Math 152 - Worksheet 7

Trigonometric Integrals

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Compute $\displaystyle\int \sin^5(x)\cos^3(x)\ dx$
2. Compute $\displaystyle\int \sin^2(x)\cos^6(x)\ dx$
3. Compute $\displaystyle\int \tan^2(x)\sec^4(x)dx$
4. Compute $\displaystyle\int \tan^2(x)\sec^3(x)\ dx$
5. Compute $\int_0^1 \sin(4x)\sin(2x)\ dx$

Submission Problems

1. Compute $\displaystyle\int \csc^4(x)\cot^4(x)\ dx$. Hint: Cotangent and cosecant work like tangent and secant.
2. Compute $\displaystyle\int \cos^4(x)\ dx$

Hint #1

1. Which of the formulas for powers of sine and cosine does this fit into?
2. Which of the formulas for powers of sine and cosine does this fit into?
3. How does this fit into the powers of tangent and secant formulas?
4. How does this fit into the formulas from earlier?
5. This falls into the product with two different frequencies type of problem.

Hint #2

1. The power of cosine is odd, so we can pull off one of them to group into the du term and convert the rest into sine terms.

2. This one needs the reduction formulas. You can either convert everything into sine or cosine. Cosine is probably easier and shorter.

3. We can rewrite $\sec^4(x)$ as $(1+\tan^2(x))\sec^2(x)$ and use this $\sec^2(x)$ as part of du

4. This one is going to need reduction formulas, so convert everything to powers of secant and go from there.

5. What formula can we use here to help us integrate this?

Hint #3

1. The integral you then want to do should be $\displaystyle\int \sin^5(x)(1-\sin^2(x))\cos(x)\ dx$
2. If you convert to cosine, the integral you need to evaluate is $\int \cos^6(x) - \cos^8(x)\ dx$
3. The integral you get to here should be $\displaystyle\int (\tan^2(x) + \tan^4(x))\sec^2(x)\ dx$
4. The integral you need to compute using reduction formulas is $\int \sec^5(x) - \sec^3(x)\ dx$
5. We can use that $\sin A \sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$ to get that $\sin(4x)\sin(2x) = \frac{1}{2}\cos(2x) - \frac{1}{2}\cos(6x)$

Hint #4

1. Set $u = \sin(x)$ to solve this integral.

2. You'll need to step down from 8 to 6 to 4 until you get to 2, and then you can apply the formula for $\cos^2(x)$ to get the answer.

3. Set $u = \tan(x)$ and $du = \sec^2(x)\ dx$, then integrate.

Answers

1. $\dfrac{1}{6}\sin^6(x) - \dfrac{1}{8}\sin^8(x) + C$

2. $-\dfrac{1}{8}\cos^7(x)\sin(x) + \dfrac{1}{48}\cos^5(x)\sin(x) + \dfrac{5}{192}\cos^3(x)\sin(x) + \dfrac{15}{384}x + \dfrac{15}{768}\sin(2x) + C$

3. $\dfrac{1}{3}\tan^3(x) + \dfrac{1}{5}\tan^5(x) + C$

4. $\dfrac{1}{4}\tan(x)\sec^3(x) - \dfrac{1}{8}\tan(x)\sec(x) - \dfrac{1}{8}\ln|\sec(x) + \tan(x)| + C$

5. $\dfrac{1}{4}\sin(4x) - \dfrac{1}{12}\sin(6x) + C$