Math 152 - Worksheet 14

Infinite Series

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Determine the pattern for the series $\frac{1}{4} + \frac{1}{6} + \frac{1}{9} + \cdots$, then write out S_2, S_4, and S_6.

2. Determine if $\displaystyle\sum_{n=3}^{\infty} \frac{3}{n(n+1)}$ converges, and if so, evaluate the sum.

3. Determine if $\displaystyle\sum_{n=2}^{\infty} \frac{1}{3} 2^n$ converges, and if so, evaluate the sum.

4. Determine if $\displaystyle\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^n$ converges, and if so, evaluate the sum.

5. Determine if $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+3}{n(n+1)}$ converges, and if so, evaluate the sum.

6. Determine if $\displaystyle\sum_{n=3}^{\infty} 4\left(\frac{1}{5}\right)^n$ converges, and if so, evaluate the sum.

Submission Problems

1. Determine if $\displaystyle\sum_{n=4}^{\infty} \frac{1}{n(n-2)}$ converges, and if so, evaluate the sum.

2. Determine if $\displaystyle\sum_{n=2}^{\infty} \frac{1}{2}\left(\frac{3}{5}\right)^n$ converges, and if so, evaluate the sum.

Hint #1

1. This is a geometric series, so we need to figure out the ratio.
2. This is in the same form as the telescoping series that we discussed previously. Try to write it in a form where terms will cancel.
3. What type of series is this?
4. What type of series is this?
5. This doesn't look like any of the types of series that we know how to evaluate. What is the only other trick we have so far?
6. This is a geometric series. What is the ratio here?

Hint #2

1. Each term is $\frac{2}{3}$ times the previous one.
2. This series can be rewritten as $\sum_{n=3}^{\infty} \frac{1}{n} - \frac{1}{n+1}$.
3. This is a geometric series. What is the ratio between consecutive terms?
4. This is a geometric series. What is the ratio between consecutive terms?
5. We could try the nth term divergence test. What happens to the terms as $n \to \infty$?
6. The common ratio is 1/5, so the series will converge. How do we evaluate the sum?

Hint #3

1. Thus, the generic term in the series is $\frac{1}{4}\left(\frac{2}{3}\right)^n$ starting at $n = 0$.

2. Write out the first few terms of this series. What can you get for a general formula for S_N?

3. The common ratio here is the part raised to the n power, or 2. What does that mean for the series?

4. The common ratio here is the part raised to the n power, or $\frac{1}{3}$. What does that mean for the series?

5. If we take the limit of the terms, we get 2. What does this mean?

6. The formula for $\sum_{n=0}^{\infty} cr^n$ is $\dfrac{c}{1-r}$. That is not the exact series we have here. We need to shift the index to zero.

Hint #4

2. $S_N = 1 - \frac{3}{N+1}$

4. So, we know the series converges, and is a geometric series. The formula for the sum is then $\dfrac{c}{1-r}$ where c is the first term of the series. (Or the r^0 term if the sum starts at 0)

6. After shifting, we should get that the series is equivalent to $\displaystyle\sum_{m=0}^{\infty} \frac{4}{125}\left(\frac{1}{5}\right)^m$ by setting $m = n - 3$. We can then use this for $\frac{c}{1-r}$. Notice how the c for this new series is the first term of the original one. This is not a coincidence.

Answers

1. The series can be written $\sum_{n=0}^{\infty} \frac{1}{4}\left(\frac{2}{3}\right)^n$, and the partial sums are $S_2 = \frac{5}{12}$, $S_4 = \frac{65}{108}$, $S_6 = \frac{665}{972}$.

2. This series converges, and the value is 1.

3. This series diverges.

4. This series converges to $\dfrac{2/3}{1-\frac{1}{3}} = 1$.

5. This series diverges by the nth term divergence test.

6. The series converges to $\frac{4/125}{1-1/5} = \frac{4/125}{4/5} = \frac{1}{25}$.