Math 152 - Worksheet 13

Sequences

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Evaluate $\lim_{n\to\infty} \dfrac{n^2 - 5n}{\sqrt{3n^4 + 4n + 2}}$ or state that it diverges.

2. Evaluate $\lim_{n\to\infty} ne^{1/n}$ or state that it diverges.

3. Write out the first 5 terms of the sequence $a_n = \frac{n^2}{n!}$, starting with $n = 1$.

4. Evaluate $\lim_{n\to\infty} \dfrac{3n^2 + 5n - \sin n}{4n^2 + 1}$ or state that it diverges.

5. Evaluate $\lim_{n\to\infty} \cos\left(\dfrac{n + \ln n}{n^2 - 2}\right)$ or state that it diverges.

Submission Problems

1. Write out the first 6 terms of the sequence $b_n = n + \frac{1}{n}$.

2. Compute $\lim_{n\to\infty} \sqrt{n+3} - \sqrt{n}$. Hint: Multiply and divide by the conjugate and simplify.

Hint #1

1. This is a sequence defined by a function, so how can we compute the limit?
2. This again is a sequence defined by a function, so how can we compute the limit?
3. To write out each term of the sequence, we need to plug in consecutive values of n.
4. This is a sequence defined by a function, so we could use our methods from before. However, that $\sin n$ is problematic. Why?
5. This is a sequence inside a function, so you can separate out the function part first and just evaluate the limit of the sequence inside.

Hint #2

1. This limit is equivalent to $\lim_{x\to\infty} \dfrac{x^2-5x}{\sqrt{3x^4+4x+2}}$.

2. This limit is equivalent to $\lim_{x\to\infty} xe^{1/x}$.

3. For $n=1$, we get $a_1 = \frac{1^2}{1!} = \frac{1}{1} = 1$.

4. Since $\lim_{n\to\infty} \sin n$ does not exist, we need to be more careful with it. The rest of the sequence is fine.

5. What happens to $\lim_{n\to\infty} \dfrac{n+\ln n}{n^2-2}$? You can split this into two terms to make it easier.

Hint #3

1. We did these limits in Calc 1. This is like the highest power rule, but you have to watch out for the square root.

2. This limit is of the form $\infty \cdot 1$, so what does that mean for evaluating it?

4. If we split off the $\frac{\sin n}{4n^2+1}$ part, the rest converges to $\frac{3}{4}$ with out an issue. What happens to this term? What theorem can we use to show that this limit exists?

5. One of these terms needs a highest power rule, the other needs LHopital.

Hint #4

4. If we use the Squeeze Theorem with $\pm\frac{1}{4n^2+1}$ as our bounding sequences, we get that this last part goes to zero.

5. You should get that this limit is zero, which you then need to put inside the cosine function.

Answers

1. $\frac{1}{\sqrt{3}}$
2. This sequence diverges.
3. $a_1 = 1$, $a_2 = 2$, $a_3 = \frac{3}{2}$. $a_4 = \frac{2}{3}$, $a_5 = \frac{5}{24}$.
4. $\frac{3}{4}$
5. 1