Math 152 - Worksheet 24

Polar Coordinates

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Find the polar coordinates of the Cartesian points $(5, -5)$, $(1, \sqrt{3})$ and $(-3, -6)$ with $r > 0$ and $0 \leq \theta < 2\pi$.
2. Find the Cartesian coordinates of the polar coordinate points $\left(3, \frac{\pi}{4}\right)$, $(5, \pi)$, and $\left(4, \frac{5\pi}{3}\right)$.
3. Convert the equation $r = \frac{1}{2-\cos\theta}$ into rectangular coordinates.
4. Convert the equation $xy = 1$ into polar coordinates of the form $r = f(\theta)$.
5. If a point $P = (x, y)$ has polar coordinates (r, θ), then what are the polar coordinates of $(x, -y)$? What about $(-x, -y)$?
6. Write the equation for the circle of radius 5 centered at $(3, 4)$ in polar coordinates. This should be of the form $r = a\cos\theta + b\sin\theta$.

Submission Problems

1. Convert the equation $(x + 2)^2 + y^2 = 4$ into polar coordinates of the form $r = f(\theta)$
2. Convert the equation $r = 2\sin\theta\tan\theta$ into rectangular coordinates. Hint: The end result here should be $y^2 =$ an expression in terms of x.

Hint #1

1. We have standard formulas for how to convert between these formulas.
2. We can again apply the standard formulas for converting these points into rectangular coordinates.
3. We do not want to start converting right away, and should look for good terms first. We can start by clearing the denominator to the other side.
4. When going to polar coordinates, we can just convert directly by plugging in our definitions.
5. If we want to analyze the point $(x, -y)$, this involves reflecting over the x-axis. How can we compare the polar coordinates of these points?
6. What is the Cartesian form of this equation?

Hint #2

1. We know that $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$, but we need to be careful about the quadrant for θ.

2. We have that $x = r\cos(\theta)$ and $y = r\sin(\theta)$

3. This gives $2r - r\cos\theta = 1$. We know that $r\cos\theta$ is x, so we can replace that and move it to the other side. We still have to deal with that r though.

4. Since we have $x = r\cos\theta$ and $y = r\sin\theta$, this equation becomes $r\cos\theta r\sin\theta = 1$.

5. To get to the opposite y-coordinate, we could rotate around in the other direction. This means I want to take the negative angle, instead of the positive one.

6. This equation is $(x-3)^2 + (y-4)^2 = 25$. We can plug in the polar coordinate versions of x and y and expand everything out.

Hint #3

1. When you take inverse tangent, you get an angle between $-\pi/2$ and $\pi/2$, and then need to figure out how to get this into the correct quadrant and the correct range of θ.

3. Once we are at $2r = 1 + x$, we can square both sides to get that $4r^2 = (1 + x)^2$, which can then be converted and simplified.

4. This becomes $r^2 = \dfrac{1}{\cos\theta \sin\theta}$ or $r^2 = \frac{2}{\sin 2\theta}$.

5. For $(-x, -y)$, this is in the opposite direction of (x, y). How do we get that in polar coordinates?

6. Expanding out gives $r^2 \cos^2\theta - 6r\cos\theta + 9 + r^2 \sin^2\theta - 8r\sin\theta + 16 = 25$.

Hint #4

3. We then have $4x^2 + 4y^2 = x^2 + 2x + 1$ or $3x^2 - 2x + 4y^2 = 1$, which we can leave as is, or simplify further by completing the square.

4. Then we can take square roots, but be careful where the function is defined.

5. For this, we can either take a negative radius, or we can add π to the angle.

6. By cancelling the coefficients and combining the cosine squared and sine squared terms, we get $r^2 - 6r\cos\theta - 8r\sin\theta = 0$

Answers

1. These points are at $\left(5\sqrt{2}, \frac{7\pi}{4}\right)$, $\left(2, \frac{\pi}{3}\right)$, and $(\sqrt{45}, \tan^{-1}(2) + \pi)$.
2. These points are at $\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$, $(-5, 0)$ and $(2, -2\sqrt{3})$.
3. $3x^2 - 2x + 4y^2 = 1$ or $3(x - \dfrac{1}{3})^2 + 4y^2 = \dfrac{4}{3}$
4. $r = \sqrt{\dfrac{2}{\sin 2\theta}}$ for $0 \leq \theta \leq \pi/2$ and $\pi \leq \theta \leq \frac{3\pi}{2}$.
5. The point $(x, -y)$ has polar coordinates $(r, -\theta)$ and the point $(-x, -y)$ has polar coordinates $(r, \theta + \pi)$.
6. $r = 6\cos\theta + 8\sin\theta$