Math 152 - Worksheet 12

Numerical Integration

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Calculate M_6 for the integral $\int_{-1}^{2} e^{x^2}\,dx$. This should be a numerical answer (calculator required).
2. Calculate T_5 for the integral $\int_{1}^{2} \sqrt{x^4+1}\,dx$ (calculator required).
3. Calculate S_6 for the integral $\int_1^4 e^{-x}\,dx$ and compare to the actual value of the integral.
4. Find the smallest value of N for with the error in the Trapezoid rule in approximating the integral $\int_3^7 \frac{1}{x}\,dx$ is less than 10^{-6}.
5. Find the smallest value of N for with the error in the Simpsons rule in approximating the integral $\int_3^7 \frac{1}{x}\,dx$ is less than 10^{-6}.

Submission Problems

1. Compute S_6 for the integral $\int_5^8 \sin\left(\frac{1}{x}\right)\,dx$.
2. Find a number N so that the error in approximating the integral $\int_2^5 5x^4 - x^5\,dx$ by M_N is less than 10^{-4}.

Hint #1

1. We need to break this into 6 rectangles. What are the endpoints and midpoints of these rectangles?

2. I want to break this into 5 trapezoids. What are the x endpoints of these trapezoids?

3. Using Simpsons rule with $N = 6$ means we need 6 rectangles that are all of width $\frac{4-1}{6} = 0.5$. What are the endpoints here?

4. We need to use the Error Bound for the Trapezoid Rule here. What do we need for this?

5. We need to use the Error Bound for the Simpsons Rule here. What do we need for this?

Hint #2

1. The midpoints are $-.75,\ -.25,\ .25,\ .75,\ 1.25,\ 1.75$.

2. Since I want 5 trapezoids, I will end up with 6 endpoints, and they should all be $\frac{2-1}{5} = 0.2$ apart. Thus, my endpoints are 1, 1.2, 1.4, 1.6, 1.8, 2.0.

3. The endpoints should be 1, 1.5, 2, 2.5, 3, 3.5, 4, which then need to be plugged into the function to apply Simpson's Rule.

4. The only tricky part here is the bound on the second derivative of the function. Since $f(x) = \frac{1}{x}$, the second derivative is $\frac{2}{x^3}$. What is the biggest this can be on the interval from 3 to 7?

5. The only tricky part here is the bound on the fourth derivative of the function. Since $f(x) = \frac{1}{x}$, the fourth derivative is $\frac{24}{x^5}$. What is the biggest this can be on the interval from 3 to 7?

Hint #3

1. Plugging these into our function will give the height, and then we need to multiply by the Δx, which is 0.5
2. We then need to plug these values into the function, and combine them appropriately for the trapezoid rule.
3. The sum you need for Simpsons Rule is $\frac{0.5}{3}\left(e^{-1}+4e^{-1.5}+2e^{-2}+4e^{-2.5}+2e^{-3}+4e^{-3.5}+e^{-4}\right)$.
4. We can use $K_2 = \frac{2}{27}$, and $b-a=4$ to solve for N in the error bound.
5. We can use $K_4 = \frac{24}{243} = \frac{8}{81}$, and $b-a=4$ to solve for N in the error bound.

Hint #4

2. The sum you should get is $0.1(\sqrt{2}+2\sqrt{1.2^4+1}+2\sqrt{1.4^4+1}+2\sqrt{1.6^4+1}+2\sqrt{1.8^4+1}+\sqrt{2^4+1})$

3. The actual integral is $e^{-1}-e^{-4}$

4. Set up $\dfrac{(2/27)4^3}{12}=10^{-6}$ and solve for N^2, then round up.

5. Set up $\dfrac{(8/81)4^5}{180N^2}=10^{-6}$ and solve for N, then round up.

Answers

3. Simpsons rule gives 0.34968, while the actual integral is 0.34956.

4. 629

5. 59