Math 152 - Worksheet 6

Integration by Parts

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Compute $\int xe^{5x}\ dx$
2. Compute $\int x^{-5}\ln x\ dx$
3. Compute $\int 3x\sin(4-x)\ dx$
4. Compute $\int 4x\cos(x^2+5)\ dx$
5. Compute $\int_0^1 x\cos\pi x\ dx$
6. Compute $\int x^2e^{2x}\ dx$
7. Compute $\int \sin^{-1}(x)\ dx$

Submission Problems

1. Compute $\int \sqrt{x}e^{\sqrt{x}}\ dx$ using any method so far. Hint: Substitution then integration by parts.
2. Compute $\int_{\pi/4}^{\pi/3} \cos(x)\ln(\sin(x))\ dx$

Hint #1

1. This is a product, so integration by parts seems to be the right way to go. What should you integrate and differentiate?

2. This is a product, so integration by parts seems to be the right way to go. What should you integrate and differentiate?

3. This is a product, so integration by parts seems to be the right way to go. What should you integrate and differentiate?

4. This one is not integration by parts. What other technique do we have so far?

5. This is a definite integral, but the idea of the problem is still the same. This is a product, so what should you differentiate and integrate?

6. If at first you don't succeed...

7. We do not have a formula for this. However, we can differentiate $\sin^{-1}(x)$. How can we use this to help us?

Hint #2

1. Try $u = x$ and $dv = e^{5x}\ dx$. What is the integral of the dv term?
2. Natural log is hard to integrate, so try $u = \ln x$ and $dv = x^{-5}\ dx$
3. Try $u = 3x$ and $dv = \sin(4 - x)\ dx$. How do you integrate this dv term?
4. Try $u = x^2 + 5$ in a u substitution
5. Try $u = x$ and $dv = \cos(\pi x)$.
6. This will require two rounds of integration by parts. Why do we know that? What's going to happen each time you do integration by parts?
7. Try $u = \sin^{-1}(x)$ and $dv = dx$.

Hint #3

1. The integral of e^{5x} is $\frac{1}{5}e^{5x}$
2. After the first round of integration by parts, you should be left with just a power of x. This can just be integrated directly.
3. The integral of $\sin(4-x)$ is $\cos(4-x)+C$.
6. You want to differentiate the polynomial part, so start with $u = x^2$. After the first round, you will want $u = x$.
7. After the round of integration by parts, you should end up with

$$x\sin^{-1}(x) - \int \frac{xdx}{\sqrt{1-x^2}}$$

Answers

1. $\frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + C$
2. $-\frac{1}{4}\frac{\ln x}{x^4} - \frac{1}{16x^4} + C$
3. $3x\cos(4-x) + 3\sin(4-x) + C$
4. $2\sin(x^2+5) + C$
5. $-\frac{2}{\pi^2}$
6. $\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$
7. $x\sin^{-1}(x) + \sqrt{1-x^2} + C$