Math 152 - Worksheet 16

Conditional Convergence and Alternating Series

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Determine if $\sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{e^n}$ converges absolutely, converges conditionally, or diverges.

2. Determine if $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^{8/9}}$ converges absolutely, converges conditionally, or diverges.

3. Determine if $\sum_{n=2}^{\infty} \frac{\sin\left(\frac{n\pi}{7}\right) n}{\sqrt{n^2+1}}$ converges absolutely, converges conditionally, or diverges.

4. Determine if $\sum_{n=4}^{\infty} \frac{(-1)^n}{n+\frac{1}{n}}$ converges absolutely, converges conditionally, or diverges.

5. How many terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ do I need to approximate the sum of the series with an error at most 0.01?

Submission Problems

1. Determine if $\sum_{n=2}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$ converges absolutely, converges conditionally, or diverges.

2. How many terms do we need to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ to within 10^{-9}?

Hint #1

1. What do the absolute value of the terms of this series look like?

2. What do the absolute value of the terms of this series look like?

3. What happens to the terms of this series as $n \to \infty$? What happens if you ignore the sine term?

4. What do the absolute value of the terms of this series look like?

5. This is an alternating series, so we can use the error bound for alternating series to work this out. What does this error bound say?

Hint #2

1. After taking the absolute value, we get the series $\sum_{n=3}^{\infty} \frac{1}{e^n}$. How can we determine if this series converges?

2. After taking the absolute value, we get the series $\sum_{n=3}^{\infty} \frac{1}{n^{8/9}}$. How can we determine if this series converges?

3. Without the sine part, the terms go to 1. What does that mean if you factor in the sine part?

4. After taking the absolute value, we get the series $\sum_{n=3}^{\infty} \frac{1}{n + \frac{1}{n}}$. How can we determine if this series converges?

5. The error bound tells us that $|S_N - S| \leq b_{N+1}$ where b_{N+1} is the first term that we leave off of the series. How does this help us?

Hint #3

1. Try the integral test on this series.

2. By p-series, the series of absolute values diverges. What about the original series?

3. If you add in the sine part, it means this limit does not exist. In particular, the limit does not equal zero. What does that mean for the series?

4. Use the fact that for $n \geq 4$, $\frac{1}{n} \leq n$ to say that $\frac{1}{n+\frac{1}{n}} \geq \frac{1}{n+n} = \frac{1}{2n}$. What do we know about this series?

5. To keep the error within 0.01, we need that $b_{N+1} = \frac{1}{\sqrt{N+1}} \leq 0.01$. We solve this out for N

Hint #4

2. Does the alternating series test apply to the original series? If so, what does that mean about its convergence?

4. Since that diverges, direct comparison says that the absolute value series diverges. Does the Alternating Series test apply to this series? What does that mean?

5. Thus $\sqrt{N+1} \geq 100$, so $N \geq 100^2 - 1$

Answers

1. $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{e^n}$ converges absolutely.

2. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n^{8/9}}$ converges conditionally.

3. $\displaystyle\sum_{n=2}^{\infty} \frac{\sin\left(\frac{n\pi}{7}\right) n^2}{(n-1)(n+3)}$ diverges by the nth term divergence test.

4. $\displaystyle\sum_{n=4}^{\infty} \frac{(-1)^n}{n+\frac{1}{n}}$ converges conditionally using the alternating series test.

5. We need to take $N \geq 9999$.