Math 152 - Worksheet 2

Area Between Two Curves

Learning Problems

These problems should be completed on your own. If you need hints on solving a problem, there are some provided with each problem. These are provided on the following pages, with one 'level' of hint per page, with the earlier ones giving away less of the problem than the later ones. Try to work from the earlier hints to the later ones, as this will give you the practice you need to succeed in this class.

1. Find the area between the curves $y = \sin(x)$ and $y = x + 1$ over the interval $[0, \pi]$.
2. Find the area of the region bounded by the curves $y = 2x + 1$ and $y = 5 + 2x - x^2$.
3. Find the area of the region bounded by the curves $y = x^2$, $y = 1$, and $y = 6 - x$ that contains the point $(2, 2)$.
4. Find the area of the region bounded by the curves $x = y^2 - 2y - 1$ and $x - 2y = 4$.
5. Find the area bounded between the curves $y = 2x$, $y = x^2 + 2x$ and $x + y = 8$.

Submission Problems

1. Find the area bounded by the curves $y = x^2 - 5$ and $y = 3 - 6x - x^2$.
2. Find the area of the triangle defined by the equations $y = x$, $y = 5x$ and $y = 12 - 3x$

Hint #1

1. Sketch a graph of these two functions. Do these curves cross?

2. Where do these curves cross? You should end up getting two points. Then which function is on top?

3. Sketch out a picture. Is this region vertically simple? What about horizontally simple?

4. Figure out where these two curves cross. The first function should indicate how you want to carry out this integral.

5. Sketch a picture and see what this region looks like. Where do all of these curves intersect?

Hint #2

1. Plugging in 0 or 1 tells us that $x + 1$ is on top and $\sin x$ is on bottom. What should the integral for the area look like?

2. The curves cross at 2 and -2, and the parabola is on top over this domain. Use that to set up the integral.

3. Figure out where these curves cross. You can either set up the problem as a single integral in y, or two integrals in x.

4. Solving for x in each expression and setting them equal gives that they cross at $y = 5$ and $y = -1$.

5. You should get intersections points of $(0, 0)$, $(\frac{10}{3} \frac{20}{3})$ and $(2, 8)$.

Hint #3

1. The integral should be $\int_0^\pi (x+1) - \sin x \; dx$
2. The integral should be $\int_{-2}^{2} (5+2x-x^2) - (2x+1) \; dx$
3. If you want to solve it as an integral in y, remember to solve out for x as a function of y in each case.
4. Which function is farther right, and thus more positive? Plug in 0 to find out.
5. You can set this area up as either two integrals in y or two integrals in x. Either way it will require two integrals, but solving out for x in the one expression is kind of annoying.

Hint #4

3. The integral you get should either be $\int_1^2 x^2-1\,dx+\int_2^5 (6-x)-1\,dx$ or $\int_1^4 (6-y)-\sqrt{y}\,dy$

4. The integral should come out to $\int_{-1}^{5} (2y+4)-(y^2-2y-1)\,dy$.

5. The integrals should be $\int_0^2 (x^2+2x)-2x\,dx+\int_2^{10/3} (8-x)-x\,dx$ or $\int_0^{20/3} (8-y)-\frac{y}{2}\,dy+\int_{20/3}^{8} (8-y)-(\sqrt{y+1}-1)\,dy$

Answers

1. $\dfrac{\pi^2}{2} + \pi - 2$
2. $\dfrac{32}{3}$
3. $\dfrac{35}{6}$
4. 36
5. $\dfrac{56}{9}$