

Arc Length and Surface Area

Learning Goals

- Setup and evaluate an x or a y integral for the length of a given curve
- Setup and evaluate an x or a y integral for a surface of revolution about the x or the y -axis

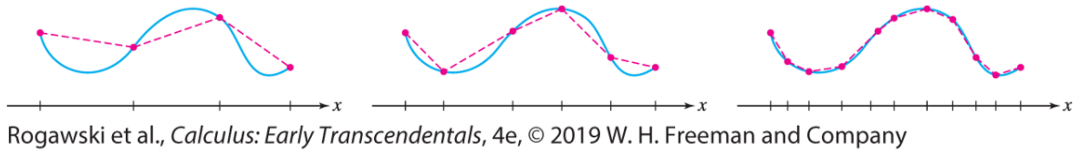
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1 Arc Length

How can we approximate the length of a curve? How did we find the area of a region?

We want to do the same thing for length: Break it up into little pieces that we know how to compute and add them up, resulting in an integral.



Each of these parts are straight lines. If this is the graph of $f(x)$, then we can find the length of each of these segments.

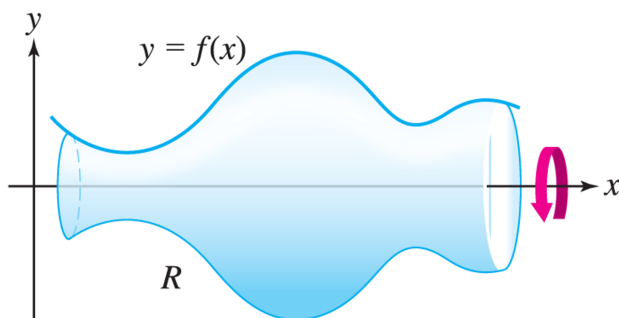
Formula for Arc Length

Assume that f' exists and is continuous on $[a, b]$. Then the arc length s of $y = f(x)$ over $[a, b]$ is equal to

Example: Find the length of the curve $y = x^{3/2}$ over the interval $[2, 5]$.

2 Surface Area

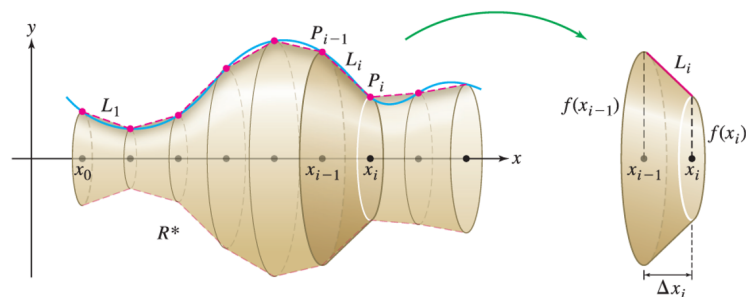
Another way we can use this idea is in computing the surface area of solids of revolution. Consider a curve $f(x)$ that we want to rotate around the x -axis.



Rogawski et al., *Calculus: Early Transcendentals*,
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How would we compute the volume?

How can we use this idea for surface area?



Rogawski et al., *Calculus: Early Transcendentals*, 4e, © 2019 W. H. Freeman and Company

Area of a Surface of Revolution: Assume that $f(x) \geq 0$ and that f' exists and is continuous on the interval $[a, b]$. The surface area S of the surface obtained by rotating the graph of f around the x -axis for $a \leq x \leq b$ is equal to

Example: Find the surface area of the surface of revolution created by revolving the graph of $y = e^{-x}$ over the interval $[2, 4]$ around the x -axis.

3 More Examples

Example: Find the length of the curve $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ over the interval $[1, 4e]$

Example: Find the arc-length of the curve $y = \frac{1}{x}$ on the interval $[2, 4]$

4 Gabriel's Horn Example

Example: Gabriel's Horn. Consider the surface generated by rotating the graph of $y = \frac{1}{x}$ on $[1, \infty)$ around the x-axis. Find the volume of this surface of revolution and its surface area.