Numerical Integration

Learning Goals

- Approximate a definite integral using either the midpoint rule, the trapezodial rule or Simpson's rule for a given N
- Compare approximation of a definite integral using either the midpoint rule, the trapezodial rule or Simpson's rule for a given N to the accurate value
- Determine error bounds for either the midpoint rule, the trapezoidal rule or the Simpson's rule
- Determine N for which the error bound for an approximation of a given integral by either the midpoint rule, the trapezoidal rule or the Simpson's rule is given
- Find the constants in an error bound for either the midpoint rule, the trapezoidal rule or the Simpson's rule

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1 Why numerical integration?

Numerical integration is important for several reasons:

1. Only an approximation to the value of an integral is needed.
2. We can't integrate a function by hand.

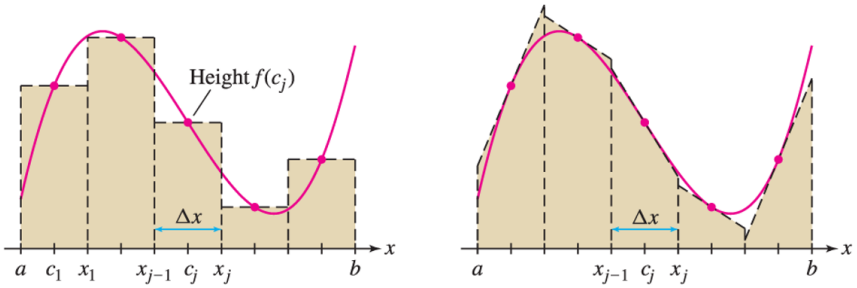
What is the idea?

We have already thought about integrals as limits of Riemann sums.

2 Midpoint and Trapezoid Rule

These are our first two rules for numerical integration. They give us formulas for finding the approximate area, given the function an interval we want to integrate over.

Midpoint Rule



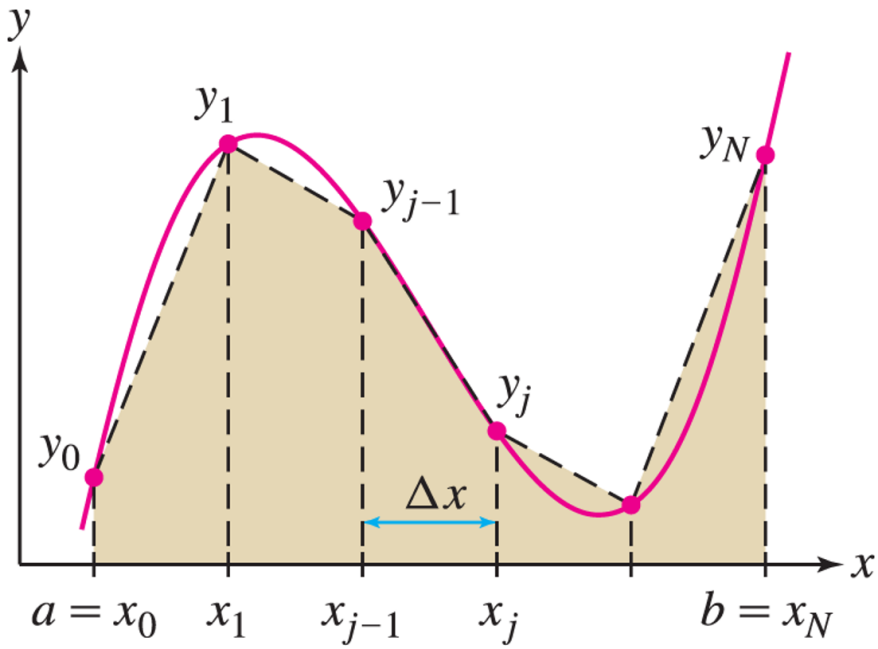
(A) M_N is the sum of the areas of the midpoint rectangles.

(B) M_N is also the sum of the areas of the tangential trapezoids.

Rogawski et al., *Calculus: Early Transcendentals*, 4e, © 2019 W. H. Freeman and Company

Formula

Trapezoid Rule



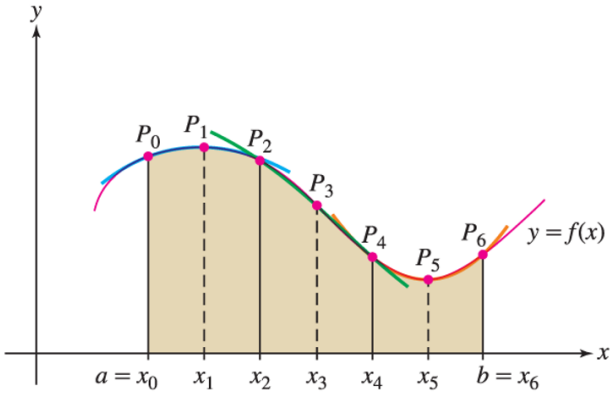
Rogawski et al., *Calculus: Early Transcendentals*, 4e, © 2019 W. H. Freeman and Company

Formula

Example: Compute M_4 and T_4 for the function $f(x) = x^2$ on the interval $[0, 4]$.

3 Simpson's Rule

What if I want to do better than straight lines between the points? This thought leads to Simpson's Rule.



Rogawski et al., *Calculus: Early Transcendentals*, 4e, © 2019 W. H. Freeman and Company

Formula

Example: Compute S_6 for the function $y = x^2$ on $[0, 6]$.

4 Error Bounds

When doing numerical integration, it's important to know how accurate your approximations are. This is useful in applied settings, when you need to be accurate within a specific amount for calculations to work.

What is an Error Bound?

Error Bound Formulas

Example: I want to use numerical integration to compute $\int_0^2 \sqrt{1+x^2}\, dx$. How many terms would I need to guarantee that my approximation was within 0.01 using all three rules?