Improper Integrals

Learning Goals

- Identify an integral as an improper integral
- Determine if an improper integral is convergent or divergent by evaluating
- Determine if an improper integral is convergent or divergent using p-integrals
- Determine if an improper integral is convergent or divergent using the comparison test

Contents

1 Definition

All of the integrals we have dealt with so far represented signed areas/volumes of bounded regions, whether that be area under a single curve, area between two curves, or volumes of revolution. What about unbounded regions? Can we talk about their areas and volumes in the same way? This gives rise to the study of **improper integrals**.

Two types of improper integrals

Convergence and Divergence

2 Infinite Intervals

How do we analyze $\int_a^\infty f(x)\ dx$?

Example: Evaluate $\int_2^\infty e^{-4x}\ dx$

3 Unbounded Integrands

What happens if the integrand goes to infinity? We think about it the same way.

Example: Calculate $\displaystyle\int_1^5 \frac{1}{(x-1)^{2/3}}\,dx$ and $\displaystyle\int_1^5 \frac{1}{(x-1)^{8/3}}\,dx$

4 The p-integral and the Comparison Test

The p-integral is defined as $\int \frac{1}{x^p}\, dx$. What regions would cause this to be an improper integral? When does it converge?

Comparison Test

Example: Does $\displaystyle\int_2^\infty \frac{1}{\sqrt{x}+e^{2x}}\,dx$ converge?

5 Multiple Issues

If there are multiple reasons or places that an integral becomes improper, you have to take care of each one separately.

Example: Evaluate $\displaystyle\int_{-2}^{3} \frac{1}{x^{-5}}\, dx$

Example: Calculate $\int_{-2}^{1} \frac{1}{x^{2/5}}\, dx$

Example: Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx$.