Trigonometric Integrals

Learning Goals

- Evaluate an indefinite or definite integral of product of trigonometric functions
- Evaluate an indefinite or definite integral of a unique trigonometric function to a power of a constant, including $\sec^3$
- Evaluate an indefinite or definite integral of product of trigonometric functions with different angles
- Find reduction formulas for integrals containing trigonometric functions to a power of n

Contents

1 Trigonometric Identities

For this section, we need to recall some important trigonometric identities from previous classes. We will need these to help us evaluate integrals.

Pythagorean Identities

Half-Angle Formulas

Product to Sum Formulas

There is another set of identities that are rarely used, but do appear in methods of integration.

$$\sin A \cos B = \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B)$$
$$\sin A \sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$
$$\cos A \cos B = \frac{1}{2}\cos(A+B) - \frac{1}{2}\cos(A-B)$$

2 Trigonometric Integrals by Substitution

So, what are the integrals that we want to investigate here? The theme of this set is integrals of the form

$$\int \sin^m x \cos^n x \; dx$$

Example: Compute $\displaystyle\int \sin^4 x \cos x \; dx$

Example: Evaluate $\int \sin^4 x \cos^3 x \; dx$.

3 Trigonometric Integrals by Reduction Formulas

There are some choices of m and n in

$$\int \sin^m x \cos^n x \; dx$$

that we can not handle yet.

Example: Evaluate $\displaystyle\int \sin^6 x \cos^4 x \; dx$

Example: Evaluate $\int \sin^2 x \; dx$

So, if we could come up with a way to reduce higher powers of sine down to squared, then we could solve the problem. This is where reduction formulas come in to play.

Reduction formula:

$$\int \sin^n(x)\ dx = -\frac{1}{n}\sin^{n-1}(x)\cos x + \frac{n-1}{n}\int \sin^{n-2} x\ dx$$

This lets us solve the problem.

Example: Evaluate $\int \sin^6 x \cos^4 x\ dx$

4 Derivation of Reduction Formulas

All of these reduction formulas are derived from integration by parts.

Example: Find a reduction formula for $\int \sin^n(x)\, dx$

Example: Find a reduction formula for $\int \sec^n(x)\,dx$

5 Integrals of Tangent and Secant

A lot of the same techniques that were applied with integrals of the form

$$\int \sin^m x \cos^n x \ dx$$

can also be applied to integrals of the form

$$\int \tan^m x \sec^n x \ dx$$

What are the 'easy' integrals of this form?

Example: Evaluate $\int \sec x \; dx$.

Example: Evaluate $\int \tan^4(x)\sec^4(x)\ dx$

Example: Evaluate $\int \tan^2(x)\sec(x)\ dx$

6 Integrals of Products with Different Frequencies

Now, we deal with more integrals of trigonometric functions, but of a different sort.

Example: Evaluate $\int \sin(4x)\cos(3x)\ dx$

The other formulas from the beginning of these notes can be used to compute integrals like

$$\int \sin(5x)\sin(3x)\ dx$$

and

$$\int \cos(2x)\cos(7x)\ dx$$