

Setting Up Integrals

Learning Goals

- Determine when certain quantities can be expressed as integrals
- Find the volume of an object given the cross-sectional area
- Find the volume of simple geometric objects using integrals
- Compute mass of objects given linear density
- Find the average value of an integrable function on an interval

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1 Applications of Integrals

Some known applications of integrals:

- Net change from rate of change
- Area between curves

What else can be done with integrals?

Examples of this:

Warning about Units:

For these problems, units can also be analyzed to determine how to solve it. It's a helpful tool, but care needs to be taken in how it is applied.

2 Volume of Simple Objects

The first application discussed here will be computing the volume of a solid from the cross-sectional area.

What happens if the area is not constant?

Units

Why does this make sense from a unit context?

Example: Find the volume of a pyramid with square base of height 5 m and base side length 6 m.

3 Volume from Cross-Sectional Area

How else can solids be described for this method?

Things to keep track of:

Example: Compute the volume of the solid whose base is the triangle enclosed by $x + y = 1$, the x -axis, and the y -axis. The cross sections perpendicular to the y -axis are semicircles.

4 Mass of Objects

Integrals can also be used to find masses of objects with varying density. The idea is the same as the volume from before.

Example: Assume that a 2m long metal rod has density given by $5(x-1)^2+2$ where x is distance in meters from one end of the rod. What is the total mass of this rod?

5 Average Value of Functions

One last application of integrals is to average value of functions. What is the average value of a set of numbers?

Formula for Average value of a function

Why is this an average value?

Mean Value Theorem for Integrals

Theorem

Example: Find the average value of the function $f(x) = x^2 - 2$ on $[0, 3]$.