Polar Coordinates

Learning Goals

- Locate points in a plane by using polar coordinates
- Convert coordinates from polar form to rectangular form and vice versa
- Convert a Cartesian equation to polar form and vice versa
- Graph polar equations by plotting points and find zeros and maximum values for a polar equation

Contents

1 Polar Coordinates

Polar coordinates give a new way to interpret equations or graphs that may make it easier to analyze. The new set of coordinates (r, θ) is defined as follows:

1. r is distance of the point from the origin
2. θ is the angle that the line from the point to the origin makes with the positive x axis in the counterclockwise direction.

Example: What are the polar coordinates of the point $(x, y) = (3, 3)$? What are the rectangular coordinates of the point $(r, \theta) = (4, \pi/3)$?

2 Conversion Formulas

How do we get between the different coordinate systems?

In general, we assume that r and θ can be any real numbers. This means that the expression of a given point in the plane is not unique.

Comparing Coordinate Systems

To compare the systems, let's think about what happens in each system when one variable is held constant.

Example: What is the polar coordinate representation of $(1, -3)$. Find at least 3 different ways to represent this point.

3 Polar Equations

When we want to describe curves in polar coordinates, we generally try to do so in the form $r = f(\theta)$. To do this, we can try to use the ideas of polar coordinates directly, or use our conversion formulas to convert an equation involving x and y, to one in terms of r and θ.

Example: Find the equation of the line $y = 2x$ in polar coordinates.

Example: Find the equation of the line $y = 3 - 4x$ in polar coordinates.

4 Converting Equations from Polar

There are a few things to keep in mind when converting equations from polar to Cartesian variables.

Example: Find the rectangular equation corresponding to the polar equation $r = 4\sin\theta$.

5 Graph Sketching

Polar graphs can be sketched in the same way as rectangular ones; plotting points and connecting them. The plotting part just needs to be interpreted in the correct way.

Example: Sketch the graph of $r = 1 + \sin\theta$.