Arc Length and Speed for Parametric Equations

Learning Goals

- Find speed of a particle moving on a parametric curve
- Find the arc length of a curve defined by parametric equations
- Find the surface area of a volume of revolution generated by revolving a parametrically defined curve about the x-axis or y-axis

Contents

1 Arc Length for Parametric Curves

Now, we want to deal with arc length, speed, and surface area for parametric curves. How can we figure out how long curves are when expressed this way?

Recall: Formulas for arc length and surface area for functions.

How did we calculate arc length before?

What does this look like for these equations?

Theorem. *Let* $c(t) = (x(t), y(t))$ *be a parametrization that directly traverses* C *for* $a \leq t \leq b$*. Assume that* $x'(t)$ *and* $y'(t)$ *exist and are continuous. Then the arc length* s *of* C *is equal to*

What do we mean by **directly traverses**? Why is this important?

Example: Find the length of the curve $x(t) = 3t^2$, $y(t) = 4t^3$ from $1 \leq t \leq 4$.

2 Distance Travelled vs. Displacement

What is the difference between these two things?

Example: Consider the path $c(t) = (t^2, e^t)$. What is the speed at $t = 1$? What is the distance travelled and displacement between $t = 1$ and $t = 3$?

3 Surface Area

We did surface area before by adding up the boundary area of little cylinders. We can do the same thing here.

Theorem. *Let $c(t) = (x(t), y(t))$, where $y(t) \geq 0$, $x(t)$ is increasing, and $x'(t), y'(t)$ are continuous. Then the surface obtained by rotating $c(t)$ about the x axis for $a \leq t \leq b$ has surface area*

Example: Find the surface area of the surface generated by revolving $c(t) = (\cos^3(t), \sin^3(t))$ for $0 \leq t \leq \frac{\pi}{2}$.