Parametric Equations

Learning Goals

- Parameterize a curve
- Find the parametric equations for a line segment given an orientation
- Eliminate the parameter in linear, polynomial, radical, exponential, logarithmic, or trigonometric equations
- Graph parametric equations by plotting points
- Find the derivative of a curve defined by parametric equations
- Find the equation of a line tangent to a parametrically defined curve

Contents

1 Definition of Parametric Equations

Imagine a particle moving along a curve C in the plane. To express this mathematically, we want to write the position of the particle as a function of time, and we will do this with two functions $x(t)$ and $y(t)$ for the x and y coordinate respectively.

We can also write this as **parametric equations**

Example: Sketch the curve given parametrically by

$$x = 4 - t \qquad y = t^2 + 2t$$

2 Eliminating the Parameter

When you want to sketch out the graph of a parametric curve, or figure out how to deal with these functions, the easiest way to do it is by trying to eliminate the parameter.

Example: Figure out the relation between x and y for the parametric equation

$$x = 4 - t \qquad y = t^2 + 2t$$

Another way this can be done is by trying to find a relation between x and y based on the equations given for them.

Example: Figure out the relation between x and y for the parametric equation

$$x = 2 + 3\cos\theta \qquad y = 1 + 3\sin\theta$$

3 Multiple Parametrizations

For any curve C, there are many ways to write a function $c(t)$ so that the particle moves along the curve. Sometimes, we refer to $c(t)$ as a *path*, which indicates that it's not just the curve C but also the way the particle moves along the path.

Example: Consider the following three parametrizations. What curve do they trace out? How do they move along this curve?

$$c_1(t) = (t, 2t) \qquad c_2(t) = (t^2, 2t^2) \qquad c_3(t) = (\sin(t), 2\sin(t))$$

4 Tangent Lines to Parametric Curves

With a curve like this, what is the slope of the tangent line?

Example: Consider the parametric curve $c(t) = (t^2 - 9, 8t - t^3)$. Find an expression (in terms of t) for $\frac{dy}{dx}$. When is the tangent line horizontal? When is it vertical?

5 Area under a parametric curve

If we have $y = f(x)$, we know how to find the area between the graph and the x-axis.

How can we do this if we have a parametric curve instead?

Example: Find the area contained inside the loop of the graph

$$c(t) = (1 - t^2, t^3 - 4t).$$