Conditional Convegence and Alternating Series

Learning Goals

- Understand the difference between absolute and conditional convergence
- Determine whether a series converges absolutely or conditionally
- Identify a series as an alternating series
- Use the Alternating Series Test to determine if a series converges
- Determine how many terms are needed to accurately approximate the sum of an alternating series

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1 Absolute and Conditional Convergence

Now, we want to start looking an series that don't necessarily have positive terms. Being able to handle series like this is really important for dealing with Taylor Series. We need a few definitions in order to handle these series.

Definition. We say that the series $\sum a_n$ **converges absolutely** if

Example: Does $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^4}$ converge absolutely? What about $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$?

Theorem. *If* $\sum |a_n|$ *converges, then* $\sum a_n$ *also converges.*

But, this is not the only way to get convergence for a series that has terms that could be positive or negative. This gives rise to another definition.

Definition. An infinite series $\sum a_n$ **converges conditionally** if

This part is hard, and the rest of this section covers a way to know if a series converges conditionally.

2 Alternating Series

For series with not necessarily positive terms, there aren't too many ways to determine if the series converges conditionally. There are a few methods that we will not discuss in this class, but the main one that we will discuss here involves alternating series.

Definition. A series is alternating if it is of the form

Examples:

Theorem. ***Alternating Series Test*** *Assume that* $\{b_n\}$ *is a positive sequence that is decreasing and converges to zero:*

Then, the following alternating series converges:

Furthermore, if S *is this sum, then*

Example: Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$. Does this converge? What can we say about the limit?

3 Proof of Alternating Series Test

What is the idea of the proof?

What is the point of this?

Example: Show that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$ converges conditionally. Furthermore if S is the sum, then $-1 \le S \le 0$.

4 Error Bound on Alternating Series

While the Alternating Series Test doesn't give us a way to compute the sums of series, it says they converge, but can get us pretty close.

Corollary. *Let* $S = \sum_{n=1}^{\infty}(-1)^{n-1}b_n$ *where* $\{b_n\}$ *is a positive decreasing sequence that converges to* 0*. Then*

Example: Analyze $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$. Does it converge? What else do we know about it? How close is S_{10} to the limit S?

Example: Determine convergence or divergence of the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}} \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$$