Series with Positive Terms

Learning Goals

- Identify a series as one with positive terms
- Determine convergence or divergence of a p-series
- Use the integral test to determine convergence or divergence of a series
- Use the Direct Comparison Test to determine convergence or divergence of a series
- Use the Limit Comparison Test to determine convergence or divergence of a series

Contents

1 Series with Positive Terms

In the last section, we talked about series and how to evaluate them. The only two tricks we really have for this is telescoping series or geometric series. However, not all series can be evaluated directly, but we still want to know if they converge or diverge. This section starts our discussion of ‘Convergence Tests’, determining whether or not a series converges without needing to compute the actual value.

Series with Positive Terms

Why start here?

What do we know if $a_n > 0$?

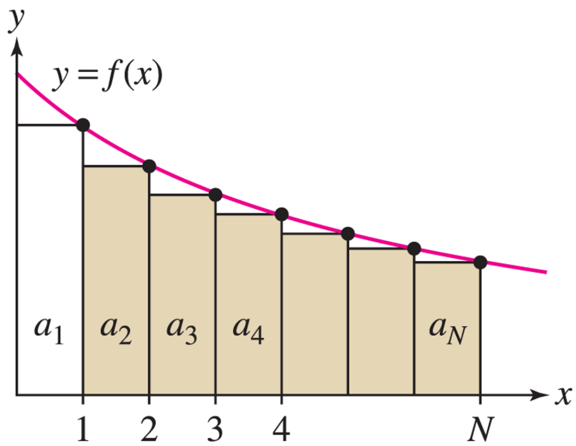
Theorem 1

2 Integral Test and p-series

Now, we want to develop some convergence tests using the result from the last video.

Theorem 2: *Integral Test*

Proof:



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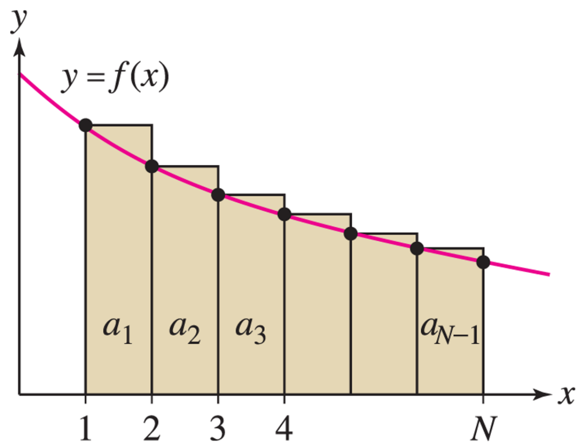

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Convergence of p-series

Example: Show that $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges but $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges.

3 Direct Comparison Test

Theorem 4: *Direct Comparison Test*

Example: Does $\sum_{n=1}^{\infty} \frac{1}{n^3+1}$ converge?

4 Limit Comparison Test

Theorem 5: *Limit Comparison Test*

Example: Determine if $\sum_{n=1}^{\infty} \frac{n^2+2}{n^3+2n^2-3n+4}$ converges.