Infinite Series

Learning Goals

- Determine whether a series converges or diverges using the sequence of partial sums
- Evaluate a convergent series using algebraic properties
- Determine if a geometric series converges and if so find its sum
- Express repeating decimals as fractions using geometric series
- Evaluate a telescoping series

Contents

1 Infinite Series

Sometimes we can't write down exact decimal expansions for numbers, generally because they are irrational and so there's no finite representation for them. This is things like e or π or $\sin 1$. However, all of these examples here can be represented as **infinite series**.

What does this equation mean?

Definition: *Convergence of an Infinite Series*

2 Telescoping Series

There are only a few series whose values we can actually compute. One of those is telescoping series.

Example: Investigate $\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$.

3 Geometric Series

The other main type of series where we can actually compute the value of is Geometric Series.

Example: Investigate $\sum c \cdot r^n$ for $|r| < 1$.

Example: Express 1.353535... as a fraction using a Geometric Series.

4 Limit Laws for Series

Theorem: Limit Laws for Series.

Example: Find the limit of the following series:

$$\sum_{n=0}^{\infty} 2\left(3^{-n} - 5^{-n}\right)$$

5 Divergence Test

This provides our first example of deciding whether or not a series converges or diverges without needing to compute its value.

Theorem: nth term Divergence Test

Non-Example: Investigate $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$