Sequences

Learning Goals

- Find the explicit formula for the nth term of an infinite sequence
- Determine if a sequence converges and if so find its limit
- Determine if a continuous function defined on a convergent sequence converges and if so finds its limit
- Use the Squeeze Theorem to find the limit of a sequence
- Determine if a sequence is bounded
- Use the properties of sequences to find limits of sequence given limits of other related sequences

Contents

1 Definition of a Sequence

This introduces the next unit of Calculus 2, which is that of sequences and series. This provides some of the foundation for what we have been doing previously and how we can apply these ideas to other areas.

Definition: A sequence $\{a_n\}$ is

Examples:

Definition: We say that $\{a_n\}$ **converges to a limit** L and write $\lim_{n\to\infty} a_n = L$ if

Theorem:

Example: Let $a_n = 1 + \frac{1}{n}$. What is $\lim_{n\to\infty} a_n$?

2 Geometric and Recursive Sequences

Example: Find the first 5 terms of the sequence defined by $a_1 = 2$, $a_2 = 3$ and $a_{n+2} = 2a_{n+1} - a_n$ for $n \geq 1$.

3 Limit Laws and Theorems for Sequences

All of the limit laws work the same way that they did before. We also have a version of the squeeze theorem.

Example: Find

$$\lim_{n\to\infty} \sin(n)e^{-n^2} + \left(3 - \frac{1}{n}\right)^2$$

4 Functions of Sequences

You can also apply functions to sequences, and that all works in the limit too, provided the function is continuous.

Example: For $f(x) = e^x$ and $a_n = \frac{\sin n}{n^2}$, what is

$$\lim_{n \to \infty} f(a_n).$$

5 Bounded Sequences

Theorems

Theorem 5:

Theorem 6: