Problem Set 1: Sections 1.1 and 1.3

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These problems are to be worked on in class. All groups should work on the first set of problems, then move on to the second and third sets. The second set will be done as presentations, and the third set will have a problem assigned as homework. This problem is due at the start of next class.

1 Warm-ups

Sketch a direction field for the following ODEs:

1. $y' = 3 - 2y$
2. $y' = 3 + 2y$
3. $y' = y + 2$
4. $y' = y(y - 2)$

Give the order of each of the following ODEs and say if they are linear or non-linear:

1. $t^2 \frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin(t)$
2. $\frac{d^3y}{dt^3} + y^2 = t^3$
3. $y\frac{d^4y}{dt^4} + t^2y = 4$
4. $\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 3y = 0$

In each problem below, verify that the given function is a solution to the ODE:

1. $y'' - 4y = 0$, $y(t) = e^{2t}$
2. $2t^2y'' + 3ty' - y = 0$, $y_1(t) = t^{1/2}$, $y_2(t) = t^{-1}$
3. $y'' + y = \sec(t)$, $0 < t < \pi/2$, $y(t) = \cos(t)\ln(\cos(t)) + t\sin(t)$

2 Exercises

1. Problem 21(a) on page 9
2. Problem 22 on page 9
3. Problem 23 on page 9
4. Problem 24(a) on page 9
5. Problem 25(a) on page 9

6. For radioactive decay, we assume that the amount of radioactive material present will decrease at a rate proportional to how much there is. Assuming that this rate is given by a positive constant k, write a differential equation for the amount $x(t)$ of radioactive material.

7. Problem 17 on page 25

8. Problem 20 on page 25

3 Problems

For the write-up, do problems 15-20 on pages 8 and 9. These are matching problems, so it's not actually 6 problems. Make sure you explain each of your answers.