Abstracts (Workshop on Multi-time Wave Functions, Rutgers University, March 26-27, 2018)

Dustin Lazarovici: *The wave function in a relativistic world*

The wave function or quantum state is a central object in any serious quantum theory. It figures in statistical predictions, mediates a structure of entanglement, and - in so-called primitive ontology formulations of quantum mechanics - determines the evolution of the fundamental constituents of matter. It may thus be surprising that the relativistic description of the wave function is not at all clear and rarely even discussed in the contemporary literature. A natural approach, first proposed by Dirac, are the so-called multi-time wave functions. The goal of this introductory talk is to discuss the various challenges of providing a sharp formulation of relativistic quantum theory, and the role multi-time wave functions could play in overcoming some of them. The main challenges we will identify are: the formulation of consistent relativistic dynamics for the wave function, the reconciliation of relativity and nonlocality and the statistical analysis of relativistic theories.

Sascha Lill: *Consistency conditions for Multi-Time Schrödinger equations*

The time evolution of an N-particle system in the multi-time-formalism is usually given by a system of N Schrödinger equations – one for each time coordinate. These PDE's may not be simultaneously solvable since the solution might depend on the order in which the time coordinates are evolved. However, if certain consistency conditions are fulfilled, problems with simultaneous solvability may be avoided. In this talk, results of S. Petrat, R. Tumulka, D. Deckert and L. Nickel will be presented, which concern those consistency conditions. It can be shown that the conditions are necessary to obtain a unique time evolution in the multi-time-formalism and that there is a wide range of classical interaction potentials, which violate them. Furthermore, interactions induced by particle exchange fulfill these conditions, if a suitable configuration space is chosen.

Roderich Tumulka: *Multi-Time Wave Functions and Wave Functions on Spacelike Hypersurfaces*

The concept of a multi-time wave function is almost the same as that of a wave function psi_Sigma for every spacelike hypersurface Sigma, and that is almost the same as the Tomonaga-Schwinger wave function. I will discuss under which circumstances any one of these almost equivalent formulations has advantages over the others. While the multi-time wave function allows for the simplest formulation of explicit time evolution equations, the psi_Sigma framework is particularly suited for general theorems. A recent such theorem that I will discuss provides a "curved Born rule," i.e., a Born rule on arbitrary (possibly curved) spacelike hypersurfaces Sigma. It asserts that if we place detectors along Sigma, then the probability distribution on configuration space is given, in the appropriate sense, by |psi_Sigma|^2. Relevant assumptions on the time evolution between hypersurfaces include "interaction locality" (i.e., that the unitary evolution involves no interaction between spacelike separated regions) and that wave functions propagate no faster than light.

Sören Petrat: *Multi-time wave functions and quantum field theory*

Multi-time wave functions offer a framework to formulate quantum mechanics in a fully relativistic setting, not presupposing any notion of simultaneity. By considering the general question of consistency of multi-time evolution equations, one is naturally led to theories with a variable number of particles, i.e., theories with particle creation and annihilation. Thus, it is very natural to consider multi-time formulations of quantum field theory (QFT). These can be regarded as a new representation of QFT, namely the multi-time Schroedinger picture particle position representation. In this talk we discuss its relation with the Tomonaga-Schwinger formalism and quantum fields in the Heisenberg picture, and we consider some simple multi-time QFT models and show their formal consistency.

Lukas Nickel: *Solution theory of a multi-time QFT model by Dirac, Fock, Podolsky*

Dirac, Fock, and Podolsky devised a relativistic model describing a fixed number of quantum particles interacting via a second-quantized electromagnetic field in 1932. This model was formulated in the multi-time picture and marked the birth of quantum electrodynamics. In this talk, it will be explained how the model can (for a scalar field with an ultraviolet cut-off) be formulated in a mathematically rigoros way. It will be shown how existence and uniqueness of smooth solutions to the model can be proven.
This is joint work with Dirk Deckert.

Emil Yuzbashyan: *Integrability of the multi-level Landau-Zener problem and multi-time Schrodinger equations*

The Landau-Zener problem is one of the key open questions in modern condensed matter physics. The problem is to find the scattering matrix from t = - infinity to t= + infinity for a Hamiltonian H(t) = A + Bt linear in time, where A and B are N x N Hermitian matrices. The 2 x 2 problem was solved by Landau, Zener and others in 1932, while for N > 2 only ad hoc solutions for few special cases have been found over the years.

We develop a unified framework to identify integrable multi-level Landau-Zener models as well as more general time-dependent Hamiltonians and solve the corresponding non-stationary Schroedinger equation. The main idea is to embed this equation into a set of multi-time Schroedinger equations. Remarkably, we find that integrable time-dependent models map to Gaudin magnets and associated Knizhnik-Zamolodchikov equations. Among new models we identify and solve are two important many-body time-dependent Hamiltonians - the inhomogeneous Dicke model with linear drive and the BCS Hamiltonian with time-dependent superconducting interaction strength.

Lukas Nickel: *Multi-time wave functions and interior boundary conditions*

Interior boundary conditions (IBCs) specify the probability flow between two sectors of Fock space and thereby allow for a mathematical formulation of particle creation and annihilation. The concept was developed by Teufel and Tumulka in order to cure the ultraviolet-divergences of certain models in quantum field theory. This talk is about ongoing research on a model of Dirac particles in 1+1 dimensions in which, for the first time, IBCs are applied to multi-time wave functions. It will be shown how this leads to a well-defined time evolution.
This is joint work with Matthias Lienert.

Ward Struyve: *Multi-time wave equations from quantum field theory*

I will describe how multi-time equations can be obtained from a relativistic quantum field theory. I will also describe some (as yet unsuccessful) attempts to get a Wheeler-Feynman type theory from a multi-time formulation.

Matthias Lienert: *Multi-time integral equations and direct relativistic interaction at the quantum level*

Because of the presence of many spacetime points in their arguments, multi-time wave functions also provide a resource to formulate completely new types of quantum theories. This possibility has been left largely unexplored. Here, we present one way to utilize the many times to obtain a new kind of interacting quantum dynamics. Starting from the reformulation of the Schrödinger equation as an integral equation, we naturally generalize this equation to a covariant integral equation for a multi-time wave function. We demonstrate that this possibility naturally expresses interaction along light cones at the quantum level. Furthermore, we discuss the question of consistency (i.e., the existence of sufficiently many solutions) and provide the first rigorous existence results for simplified models.

Michael Kiessling: *Relativistic Quantum Mechanics of an Electron-Photon System in 1+1 Dimensional Minkowski Spacetime*

This talk summarizes the results of a joint work with Matthias Lienert and A. Shadi Tahvildar-Zadeh. A fully Lorenz-covariant system of equations for the multi-time wave function of the two-body quantum-mechanical electron-photon system in 1+1 dimensional Minkowski spacetime is formulated. Electron and photon are interacting via a Lorentz-invariant configurational boundary condition derived from particle current conservation. It is proved that the corresponding initial-boundary-value problem is well-posed. Electron and Photon trajectories are obtained via a hypersurface Bohm-Dirac-type law of motion.

Sheldon Goldstein: *What is the point of multi-time wave functions?*

I will discuss what multi-time wave functions are good for in the various versions of quantum mechanics, including Copenhagen.

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