Demetri Christodoulou
Swiss Federal Institute of Technology Zürich

Acoustical Geometry and Shock Formation

The equations describing the motion of a perfect fluid were first formulated by Euler in 1752. These equations were among the first partial differential equations to be written down, but, after a lapse of two and a half centuries, we are still far from adequately understanding the observed phenomena which are supposed to lie within their domain of validity. These phenomena include the formation and evolution of shocks in compressible fluids. The first work on shock formation was done by Riemann in 1858. However, his analysis was limited to the simplified case of one spatial dimension.

In 2007 I published a monograph which treated the relativistic Euler equations in three space dimensions for a perfect fluid with an arbitrary equation of state. In this monograph I considered initial data which outside a sphere coincide with the data corresponding to a constant state. Under a suitable restriction on the size of the initial departure from the constant state, I established theorems which gave a complete description of the maximal classical development. In particular, I showed that the boundary of the domain of the maximal classical development has a singular part where the inverse density of the wave fronts vanishes, signalling shock formation. In fact, the theorems which I established gave a complete picture of shock formation in three-dimensional fluids. In my talk I shall give a simplified presentation of these results and of their proof, assuming from the outset that the initial conditions are irrotational. The approach is geometric, the central concept being that of the acoustical spacetime manifold.