Diffusion in a periodic Lorentz gas with narrow tunnels

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Abstract

In a periodic Lorentz gas a particle moves bouncing off a regular array of fixed convex obstacles (scatterers), like in a pinball machine. When the horizon is finite, one observes a classical diffusion law. When the obstacles are so large that the tunnels between them become narrow (of width $\epsilon \to 0$) then the diffusion matrix scales with $\epsilon$. In the limit, when $\epsilon = 0$, the particle is trapped in a compact region with cusps in the boundary. In that case the system ceases to be uniformly hyperbolic and develops anomalies. Correlations decay slowly, as $1/n$, and the classical central limit theorem fails. Instead, a non-classical limit law holds, with a scaling factor of $\sqrt{n \log n}$ replacing the standard $\sqrt{n}$. However, for a special observables whose average values at the cusps vanish, the classical central limit law still holds. In collaboration with P. Balint and D. Dolgopyat.