GLOBAL CLASSICAL SOLUTIONS OF THE BOLTZMANN EQUATION WITH LONG-RANGE INTERACTIONS

ABSTRACT OF ROBERT STRAIN

ABSTRACT. In this talk we explain our recent proof of global stability for the Boltzmann equation 1872 with the physically important collision kernels derived by Maxwell 1867 for the full range of inverse power intermolecular potentials, $r^{-(p-1)}$ with p > 2 and more generally. Our solutions are perturbations of the Maxwellian equilibrium states, and they decay rapidly in time to equilibrium. This equation provides a basic example where a wide range of geometric fractional derivatives occur in a physical model of the natural world.

We additionally prove a conjecture of Mouhot-Strain 2007, showing that a spectral gap exists for the linearized Boltzmann collision operator if and only if $\gamma + 2s \ge 0$. We specifically prove sharp constructive upper and lower bounds for the linearized collision operator.

Our methods also provide a new fully non-linear coercive "entropy production" estimate in terms of the same geometric fractional semi-norm as in the linearized context. This semi-norm is sharper than previously known coercive lower bounds, it is sharper in terms of the non-isotropic weight which is fundamentally intertwined with the fractional differentiation effects.

This is joint work with P. Gressman