SOME REMARKS ON THE SURFACE TENSION.

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We consider the surface tension $\beta^{-1}\tau$ for the d-dim. Ising model with nearest neighbour ferromagnetic coupling $J > 0$. $\tau$ is defined in e.g. [1]:

$$0 \leq \tau = -\lim_{L \to \infty} \frac{1}{(2L+1)^d-1} \lim_{M \to \infty} \log \frac{Z_{\Lambda, +}}{Z_{\Lambda, -}}$$ (1)

where $\Lambda$ is a box centered at the origin of height $2M$ in the $i_1$-direction with a base of side $2L+1$. The indices $+$ and $-$ refer to the usual $+$ and $-$ boundary conditions (b.c.).

We do the following construction: we replace the couplings $J$ by $sJ$, $0 \leq s \leq 1$ for all nearest neighbour pairs $<ij>$ crossing the plane $i_1 = -\frac{1}{2}$. By symmetry $\tau(s)$ is zero for $s = 0$ and $\tau = \tau(1)$ is equal to

$$\tau = k \int_0^1 ds \left( \rho_s^+ (\sigma_0^{1-s} \Delta \sigma_0^{1-s}) - \rho_s^+ (\sigma_0^{1-s} \Delta \sigma_0^{1-s}) \right), \delta J = k$$ (2)

where $\rho_s^+$ is the corresponding infinite volume Gibbs state with +b.c. ($+b.c.$) (2) is justified by correlation inequalities and using such inequalities we prove that $\tau = 0$ whenever there is no spontaneous
magnetization. We can also prove that $\tau = 0$ if the spontaneous magnetization is zero for the semi-infinite system defined on $\{i \in \mathbb{Z}^d: i_1 > 0\}$ with free b.c. at $i_1 = 0$ and +b.c. elsewhere. From (2) and correlation inequalities we obtain the lower bound

$$\tau \geq 2k \int_0^1 \rho_s^+ (\sigma_0) \rho_s^- (\sigma_0) \, ds$$

Using duality and correlation inequalities we prove that for $d=2$ $\tau(k) = m(k_*)$ where $k_*$ is the dual temperature and $m$ is the inverse correlation length in the state with free b.c. For $d=3$ $\tau(k) = \alpha(k_*)$ where $\alpha$ is the coefficient of the area law decay of the Wilson loop in the Ising gauge model [2].

For $d=3$ and $k$ large enough we prove that $\tau - 2k$ and the correlation functions in the state $\rho^\pm$ are analytic in $\exp(-2k)$. Furthermore we have the Gibbs formula

$$\frac{d\tau}{dk} = \sum_{i_1 = -\infty}^{+\infty} \sum_{j: |i-j|=1} (\rho^+(\sigma_i \sigma_j) - \rho^-(\sigma_i \sigma_j))$$

see [2] and [3].

References:

