1. Prove Farkas’ Lemma (if you haven’t seen it), e.g. in the form:
if \( f : \mathbb{N} \rightarrow \mathbb{R} \) is superadditive (i.e. \( f(a + b) \geq f(a) + f(b) \)), then \( \lim n^{-1} f(n) \) exists (it may be infinite) and is at least \( m^{-1} f(m) \) for every \( m \).

2. As in class, let \( f(k) = f_3(k) \) be the maximum size of a \( k \)-uniform \( F \) with no sunflower, here meaning of size 3, and let \( g(k) \) be the maximum size of such an \( F \) that is also intersecting (i.e. \( A \cap B \neq \emptyset \forall A, B \in F \)).
   (a) Show that \( f(kl) \geq f(k)g(l)^k \).
   (b) Conclude that for \( k \) a power of 3, \( f(k) \geq 2g(k) \geq 2 \cdot 10^{(k-1)/2} \).
   [Hint: Start with a 3-uniform, 10-edge hypergraph gotten by identifying antipodal points of an icosahedron.]

3*. Prove the Erdős-Szemerédi Conjecture for (any) \( r \geq 4 \).

4. Prove Ellenberg-Gijswijt: \( g(n) < (2.75 \cdots)^n \) (or just \( g(n) < (3 - \varepsilon)^n \)).

5. Give an elementary (non-linear algebraic) proof of the C-D Theorem.
   [Minor suggestion: use induction on \( |B| \) (say).]

6. Prove the lemma from class that underlies the “Nullstellensatz”:
   If \( S_1, \ldots, S_n \subseteq \mathbb{F} \) (a field) and \( f \in \mathbb{F} \) vanishes on \( \prod S_i \) and is “reduced” (i.e. \( \deg_S(f) < s_i := |S_i| \forall i \)), then \( f \equiv 0 \) (coefficientwise).

7. Prove the Alon-Nathanson-Ruzsa result stated in class:
   For \( p \) prime and \( A, B \) nonempty subsets of \( \mathbb{Z}_p \) with \( a = |A| \neq |B| = b, \)
   \[ |A \oplus B| \geq \min\{p, a + b - 2\} \]
   (where \( A \oplus B = \{\alpha + \beta : \alpha \in A, \beta \in B, \alpha \neq \beta\} \)).

8*. For any \( k \) and \( n \), the “Davenport constant” of \( \mathbb{Z}_k^n \) is \( m := n(k - 1) + 1 \); that is, for any \( a^1, \ldots, a^m \in \mathbb{Z}_k^n \) there is some \( \emptyset \neq I \subseteq [m] \) with \( \sum_{i \in I} a^i = 0 \).

9. Show that one can’t cover \( \{0, 1\}^n \setminus \{0\} \) (\( \subseteq \mathbb{R}^n \)) by fewer than \( n \) affine hyperplanes not containing \( 0 \).
An affine hyperplane is \( \{ x \in \mathbb{R}^n : a \cdot x = b \} \) for some \( a \neq 0 \in \mathbb{R}^n \) and \( b \in \mathbb{R} \). Of course this fails to contain 0 iff \( b \neq 0 \). You should check that \( n \) is best possible, and see what happens if we allow the hyperplanes to contain 0.

10. Prove that (as stated in class) the graph on \( \{ k \text{-trees of } (V_r) \} \) (with \( T \sim T' \) iff \( T \cap T' \) is a \( (k-1) \)-tree) is connected.