1. Let $S$ be a finite set and $m \in \mathbb{P}$, and consider two experiments:
   (a) $x_1, \ldots, x_{2m}$ are chosen uniformly and independently from $S$;
   (b) $\{y_1, z_1\}, \ldots, \{y_m, z_m\}$ are chosen uniformly and independently from $\binom{S}{2}$.
Set $X = \{x_1, \ldots, x_{2m}\}$ and $Y = \{y_1, \ldots, y_m, z_1, \ldots, z_m\}$, and prove the rather obvious fact that $\mathbb{P}(|Y| \geq t) \geq \mathbb{P}(|X| \geq t)$ \forall t.

2. Let $\mu$ be uniform measure on $2^S$ ($S$ a finite set), let $\mathcal{A}_1, \ldots, \mathcal{A}_r$ be increasing subsets of $2^S$ with $\max \mu(\mathcal{A}_i) \leq 1/2$, and set
   $$\mathcal{A} = \{X \subseteq S : \exists i \in [r] \text{ with } X \in \mathcal{A}_i\}.$$Show $\mu(\mathcal{A}) < 1 - c$ for some fixed positive $c$.
[Open and very interesting: if we assume $\max \mu(\mathcal{A}_i) < \varepsilon$, is it true that
   $$\mu(\mathcal{A}) < e^{-1} + o(1) \text{ where } o(1) \to 0 \text{ as } \varepsilon \to 0?$$
(Why would $e^{-1}$ be best possible?)]

3. Let $P$ be a poset, $A \subseteq P$, and define $f : 2^A \to \mathbb{N}$ by
   $$f(X) = |\{I : I \text{ an ideal of } P, I \cap A = X\}|.$$Show $f$ is log supermodular (i.e. $f(X)f(Y) \leq f(X \cap Y)f(X \cup Y)$ \forall $X, Y \subseteq A$).
[Easy once found.]