Problem 1

    (1) Find the Laplace transform of

$$e^{-3t}.$$

    (2) Find the Fourier transform of

$$e^{-3|x|}.$$

Problem 2 Solve the following initial value problem

$$\begin{cases} u_t = u_{xx}, & x > 0, \ t > 0, \\ u_x(0, t) = 0, & t > 0, \\ u(x, 0) = x^2, & x > 0. \end{cases}$$

Problem 3 Solve the following initial value problem

$$\begin{cases} u_{tt} = u_{xx} - e^{-|x|}, & x > 0, t > 0, \\ u(0, t) = 0, & t > 0, \\ u(x, 0) = u_t(x, 0) = 0, & x > 0. \end{cases}$$

Problem 4 Solve the following Laplace equation.

$$\begin{cases} u_{xx} + u_{yy} = 0, & x \in \mathbb{R}, \ y > 0, \\ u(x, 0) = -3, & |x| \leq l, \\ u(x, 0) = 0, & |x| > 1. \end{cases}$$

Problem 5 Suppose $u(x, t)$ satisfies the wave equation

$$u_{tt} = u_{xx}.$$

Show that

$$v(x, t) = \frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{\infty} u(s, x) e^{-\frac{s^2}{4t}} \, ds$$

solves the heat equation

$$v_t = v_{xx}.$$