Practice final exam

Problem 1 Solve the following equation

$$\left\{ \begin{array}{ll} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, & x > 0, t > 0, \\ u(0,t) = 1, & t > 0, \\ u(x,0) = 0, & x > 0. \end{array} \right.$$

Problem 2 Solve the following wave equation

$$\left\{ \begin{array}{ll} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, & 0 < x < \pi, t > 0, \\ u(0,t) = 0, \ u(\pi,t) = 0, & t > 0, \\ u(x,0) = \frac{1}{6}x(\pi^2 - x^2), \ \frac{\partial u}{\partial t}\Big|_{t=0} = 0, & 0 < x < \pi. \end{array} \right.$$

Problem 3 Solve the following Laplace equation

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, & 0 < x < 1, 0 < y < 1, \\ u(0, y) = 10y, \ \frac{\partial u}{\partial x}\Big|_{x=1} = -1, \\ u(x, 0) = 0, \ u(x, 1) = 0. \end{cases}$$

Problem 4 Solve the following boundary value problem.

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 2u, & 0 < x < \pi, \ t > 0, \\ u(0,t) = 0, & u(\pi,t) = -1, \ t > 0, \\ u(x,0) = 0, & 0 < x < \pi. \end{cases}$$

Problem 5 Solve the following boundary value problem.

$$\begin{cases} \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0, & 0 \le \theta \le 2\pi, \ 0 < a < r < b, \\ u(a, \theta) = f(\theta), \ u(b, \theta) = g(\theta), & 0 \le \theta \le 2\pi. \end{cases}$$

Problem 6 Solve the following boundary value problem.

$$\begin{cases} u_t = u_{xx}, & 0 < x < \pi, \\ u = (0, t) = e^{-t}, \ u(\pi, t) = t, & t > 0, \\ u(x, 0) = x, & 0 < x < \pi. \end{cases}$$