1. Suppose $x$, $y$, and $z$ are implicitly related by the equation

$$x^2 y + y^2 z + xz^2 = 1$$

Calculate the derivatives of $z$ with respect to $x$ and $y$.

**Solution**

We implicitly differentiate with respect to $x$.

$$2xy + y^2 z_x + 2xz z_x + z^2 = 0$$

Solving for $z_x$ gives

$$
\frac{\partial z}{\partial x} = -\frac{2xy + z^2}{2xz + y^2}
$$

Now implicitly differentiate with respect to $y$.

$$x^2 + y^2 z_y + 2yz + 2xz z_y = 0$$

Solving for $z_y$ gives

$$
\frac{\partial z}{\partial y} = -\frac{2yz + x^2}{2xz + y^2}
$$