1. The temperature at the point \((x, y, z)\) is given by

\[ T(x, y, z) = 4 + x^3y + xz + y^2z^2 \]

A particle travels along the helix parametrized by

\[ \mathbf{r}(t) = (\cos(t), \sin(t), t) \]

What is the rate of change of the temperature along the particle’s path when \(t = 0\)?

**Solution**

We have the following.

\[ \mathbf{r}(0) = (1, 0, 1) \]
\[ \nabla T(1, 0, 1) = \left(3x^2y + z, x^2 + 2yz^2, z + 2y^2z\right) \bigg|_{(1,0,1)} = (1, 1, 1) \]
\[ \mathbf{r}'(0) = (-\sin(t), \cos(t), 1) \bigg|_{t=0} = (0, 1, 1) \]

Hence the desired rate of change is

\[ \left. \frac{dT}{dt}(\mathbf{r}(t)) \right|_{t=0} = \nabla T(\mathbf{r}(0)) \cdot \mathbf{r}'(0) = (1, 1, 1) \cdot (0, 1, 1) = 2 \]

2. At the point \(P = (-3, 4)\), the function \(f(x, y)\) has gradient \(\nabla f(P) = (1, 2)\). What is the rate of change of \(f\) at the point \(P\) in the direction 45 degrees north of west?

**Solution**

A unit vector pointing in the direction 45 degrees north of west is \(\mathbf{u} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\). Hence the desired rate of change is

\[ D_{\mathbf{u}}f_P = (1, 2) \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} \]