1. For each function $f$, describe the contour curves. You should give a complete, concise, and clear English description rather than simply graph various contour curves.

(a) $f(x, y) = 2x - y + 5$  
(b) $f(x, y) = e^{x-y^2}$  
(c) $f(x, y) = \ln(x^2 + y^2 + 9)$

Solution
(a) Putting $f(x, y) = c$ where $c$ is a constant, we see the contour curves have equations of the form $2x - y + 5 = c = \text{const}$. These are parallel lines of slope 2.
(b) Putting $f(x, y) = c$, we see the contour curves have equations of the form $x - y^2 = \ln(c) = \text{const}$. These are parabolas which open rightward and whose vertices lie on the $x$-axis.
(c) Putting $f(x, y) = c$, we see the contour curves have equations of the form $x^2 + y^2 = e^c - 9 = \text{const}$. These are circles centered at the origin.

2. Calculate the following limit or determine it does not exist.

$$\lim_{(x,y) \to (0,0)} \left( \frac{xy}{\sqrt{x^2 + y^2}} \right)$$

Solution
We consider the limit along an arbitrary path expressed in polar coordinates $x(t) = r(t) \cos(\theta(t))$ and $y(t) = r(t) \sin(\theta(t))$.

$$\lim_{(x,y) \to (0,0)} \left( \frac{xy}{\sqrt{x^2 + y^2}} \right) = \lim_{r \to 0} (r \cos(\theta) \sin(\theta))$$

Now note that $-1 \leq \cos(\theta) \leq 1$ and $-1 \leq \sin(\theta) \leq 1$. Hence we have

$$-r \leq r \cos(\theta) \sin(\theta) \leq r$$

The outer expressions have limit 0 as $r \to 0$. Hence by the squeeze theorem, we find

$$\lim_{r \to 0} (r \cos(\theta) \sin(\theta)) = 0$$