1. Let $\mathcal{D}$ be the trapezoid in the $xy$-plane with vertices $(-1, -1), (1, -1), (1, 2),$ and $(-1, 4)$. Let $\mathcal{C}$ denote the boundary of $\mathcal{D}$, oriented anticlockwise. Calculate

$$\int_{\mathcal{C}} ((x^3 - xy) \, dx + (4x + 7y) \, dy)$$

**Solution**

Note that $\mathcal{C}$ is closed and oriented anticlockwise, so we may use Green’s Theorem. The region $\mathcal{D}$ is a trapezoid with vertical left and right sides. The bottom side lies on the line $y = -1$ and the top side lies on the line $y = 3 - x$. Hence $\mathcal{D}$ can be described by the inequalities

$$-1 \leq y \leq 3 - x, \quad -1 \leq x \leq 1$$

Note that $\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 4 + x$, and so we have the following.

$$\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r} = \iint_{\mathcal{D}} (4 + x) \, dA = \int_{-1}^{1} \int_{-1}^{3-x} (4 + x) \, dy \, dx = \int_{-1}^{1} (4 + x)(3 - x - (-1)) \, dx$$

$$= \int_{-1}^{1} (16 - x^2) \, dx = \left(16x - \frac{1}{3}x^3\right)\bigg|_{-1}^{1} = \frac{94}{3}$$