1. Let $\mathbf{F} = \langle 2xy + 3yz, x^2 + 3xz, 3xy + 15z^2 \rangle$.

(a) Calculate $\nabla \cdot \mathbf{F}$.

(b) Without calculating a potential, show that $\mathbf{F}$ is conservative.

(c) Find a potential for $\mathbf{F}$.

Solution

(a) $\nabla \cdot \mathbf{F} = 2y + 0 + 30z = 2y + 30z$

(b) The domain of $\mathbf{F}$ is $\mathbb{R}^3$ (which is simply connected) and $\nabla \times \mathbf{F} = 0$.

(c) By inspection, we see that a potential is $f(x, y, z) = x^2y + 3xyz + 5z^3$. 