Big Picture and Beyond

Big Topics in This Course:

1. Vector Geometry
   - equations of line
   - equations of plane
   - dot, cross product

2. Parametrizations
   - curves (circle, line)
   - surfaces (disk, plane, cone, paraboloid, sphere)
   - interpretation of parameters

3. Constrained Optimization
   - find min/max of $f$ on some closed set $U$
   - Lagrange multipliers for finding min/max on $2U$
Vector Calculus Theorems:
- gradient
- Green
- Stokes
- divergence

The vector calculus theorems are actually all special cases of a theorem from advanced geometry called "Stokes's Theorem"

\[ \int_M \omega = \int_{\partial M} \omega \]

(Take a course in geometry.)

FTC: \[ \int_a^b f'(x) \, dx = f(b) - f(a) \]
Gradient: \[ \int_C \nabla f \cdot \, d\vec{r} = f(Q) - f(P) \]
Green: \[ \int (\partial F_x - \partial F_y) \, dx = \int (\partial F_y - \partial F_z) \, dy = \int (\partial F_z - \partial F_x) \, dz \]
Green's Theorem: \( \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dx \, dy = \oint_{\partial D} P \, dx + Q \, dy \)

Stokes' Theorem: \( \iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{r} \)

Divergence: \( \iiint_V (\nabla \cdot \vec{F}) \, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S} \)

_left side:_ integral of a derivative on some set or region \( U \).

_Right side:_ integral of the original function on \( \partial U \).

In electro dynamics, we use these theorems to find \( \vec{E} \) and \( \vec{B} \) with a lot of symmetry.

\[ \oint \vec{u} \cdot d\vec{r} = \text{circulation} \]

\( \nabla \cdot \vec{u} = 0 \) incompressible