1. Let $f(x, y) = 3x^3 - 12xy + 2y^2 + 10$. Find all critical points of $f$. Then classify each critical point as either a local minimum, local maximum, or neither (saddle).

2. The humidity at the point $(x, y, z)$ is modeled by the function $H(x, y, z) = 20e^{-z}(1 + x^2 + y^2)^{-1}$. A weather balloon is equipped with a device to measure the humidity, and the balloon travels along the path given by $\mathbf{r}(t) = (\sqrt{t}, 4\sin(\pi t), t^2)$.

   (a) Calculate $\nabla H(x, y, z)$.
   (b) Find a parametrization of the line tangent to the balloon’s path at $t = 4$.
   (c) At what rate does the balloon’s device measure the humidity to be changing when $t = 4$?

3. The planes $P_1$ and $P_2$ are described by the following equations.

   $P_1 : \ 3x - 2y + z = 14$
   $P_2 : \ 5x + y - 8z = 6$

   (a) Find the angle between $P_1$ and $P_2$.
   (b) The planes $P_1$ and $P_2$ intersect in the line $\ell$. Find a parametrization of $\ell$.

4. Find the largest possible volume of a box with one corner at the origin and the opposite corner at a point $(x, y, z)$ in the first octant $(x, y, z \geq 0)$ and lying on the paraboloid

   $z = 1 - \frac{x^2}{4} - \frac{y^2}{9}$

5. For each part, find a parametrization of the described curve. Each parametrization should consist of a single vector-valued function $\mathbf{r}(t)$.

   (a) the circle of radius 5 with center $(9, -1, 2)$ and lying in a plane parallel to the $xz$-plane
   (b) the intersection of the cone $x^2 + y^2 = z^2$ and the plane $3z = x + 4$

6. Let $f(x, y, z) = \frac{x^2y}{z}$. Use linear approximation to estimate the value of $f(2.03, 3.01, 0.98)$. Simplify your answer.

7. Assume that the positive $x$-axis points East and the positive $y$-axis points North. Suppose you are hiking on a terrain modeled by the equation $z = 3 - 4y - x^2 + 2xy$ and you are standing on the terrain above the point $P(-1, 2)$.

   (a) Determine the angle of inclination you would encounter if you were to head due North.
   (b) Determine the steepest possible slope of the terrain at $P$.
   (c) At what compass angle should you travel from $P$ to encounter the slope of steepest descent? (The compass angle should be measured anticlockwise from East.)