1. Let $W$ be a solid right circular cone with its vertex at the origin, with radius $R = 1$ and height $H = 2$. **Using spherical coordinates**, calculate the following integral.

$$\iiint_W \frac{2}{\sqrt{x^2 + y^2 + z^2}} \, dV$$

2. Let $D$ be the region in the first quadrant that is inside the larger circle $x^2 + y^2 = 36$ and outside the smaller circle $(x-3)^2 + y^2 = 9$. **Using polar coordinates**, calculate the following integral.

$$\iint_D 3\sqrt{x^2 + y^2} \, dA$$

3. Let $W$ be the region in the first octant that is bounded by the surface $z = 27 - x^3$ and the plane $x = 3y$. Write the integral

$$\iiint_W xy(x^2 + z^2) \, dV$$

as an iterated integral in the order $dydxdz$. **Do not evaluate your integral.**
4. Use the method of Lagrange multipliers to find the point on the ellipse

\[ x^2 + 6y^2 + 3xy = 40 \]

with the largest x-coordinate.

5. Consider the vector field \( F(x, y, z) = \langle 3zy - 1, 4x, -y \rangle \).

8 pts

(a) Calculate the divergence and curl of \( F \).

8 pts

(b) Let \( C \) be the curve given by \( r(t) = \langle e^t, e^t, t \rangle \) for \( 0 \leq t \leq 1 \). Calculate the line integral of \( F \) along \( C \).

6. Let \( D \) be the region in the \( xy \)-plane described and shown in the figure below.

\[ D : \quad x^2 \leq y \leq 3x^2, \quad 1 \leq x^2y \leq 4 \]

(a) Find a rectangle \( R \) in the \( uv \)-plane and a map \( G \) such that \( G(R) = D \).

You may either specify \( G \) (give \( x \) and \( y \) in terms of \( u \) and \( v \)) or specify \( G^{-1} \) (give \( u \) and \( v \) in terms of \( x \) and \( y \)).

7 pts

(b) Find \( |\text{Jac}(G)| \). You may give your answer in terms of \( x \) and \( y \) or in terms of \( u \) and \( v \).

7 pts

(c) Calculate the following integral.

\[ \iint_D 6x^3y \, dA \]