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This exam has 8 questions, printed in 2 booklet(s), for a total of 100 points.

<table>
<thead>
<tr>
<th>Question</th>
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1. Let $v = i - \lambda j + 3k$ and $w = \lambda i - 4j + \mu k$, where $\lambda$ and $\mu$ are real numbers.

(a) Calculate $\|v\|$ in terms of $\lambda$ and $\mu$.

\[ \|v\| = \text{______________} \]

(b) Calculate $v \cdot w$ and $v \times w$ in terms of $\lambda$ and $\mu$.

\[ v \cdot w = \text{______________} \]
\[ v \times w = \text{______________} \]

(c) Use your answer to part (b) to find the values of $\lambda$ and $\mu$ such that $v$ and $w$ are parallel. (Assume $\lambda$ and $\mu$ are positive real numbers.)

\[ \lambda = \text{______________} \]
\[ \mu = \text{______________} \]
2. The lines \( \ell_1 \) and \( \ell_2 \) are given by the following parametrizations.

\[
\ell_1: \quad \mathbf{r}_1(t) = \langle -2, -1, 4 \rangle + t \langle -5, 5, 1 \rangle \\
\ell_2: \quad \mathbf{r}_2(t) = \langle 0, -10, 10 \rangle + t \langle 3, 4, -7 \rangle
\]

(a) Show that \( \ell_1 \) and \( \ell_2 \) intersect and find the point of intersection. Is this point also a collision point? Explain.

point of intersection: ________________

collision point? (yes/no): ________________

(b) Find an equation of the plane \( P \) that contains both \( \ell_1 \) and \( \ell_2 \).

equation of \( P \): ________________
3. Consider the curve \( C \) with parametrization

\[
\mathbf{r}(t) = (t^2 - 3)i + (3t^2 + 5)j + \frac{2}{3}t^3k, \quad t \geq 0
\]

(a) Find the length of \( C \) over the interval \( 0 \leq t \leq 1 \).

\[
\text{length: } \quad \left[ \right]
\]

(b) Find the curvature of \( C \) at the point \( \mathbf{r}(1) \). You may use the formula

\[
\kappa(t) = \frac{\|\mathbf{r}''(t) \times \mathbf{r}'(t)\|}{\|\mathbf{r}'(t)\|^3}
\]

\[
\text{curvature: } \quad \left[ \right]
\]
4. Assume that the positive $x$-axis points East and the positive $y$-axis points North. Suppose you are hiking on a terrain modeled by the equation $z = \sqrt{3}xy - 2x^2 - 1$ and you are standing at the point $(1, \sqrt{3}, 0)$.

(a) Determine the angle of inclination you would encounter if you headed due West.

angle of inclination: ____________________

(b) Determine the steepest slope you could encounter from your position and the compass direction measured in degrees anticlockwise from East that you would head to realize this steepest slope.

steepest slope: ____________________
compass direction: ____________________

(c) In what direction should you head to encounter no change in elevation? Give your answer as an angle measured in degrees anticlockwise from East.

compass direction: ____________________
This page is for scratch work. Do not detach this sheet.
Book #2 of 2

Name: ____________________________

ID# (last 4 digits): _______________  Section: ____________________________

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<tr>
<td>Total:</td>
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</table>
5. Calculate the following limit or show that it does not exist.

\[
\lim_{{(x,y) \to (0,0)}} \left( \frac{xy}{3x^2 + 2y^2} \right)
\]

value of limit: ________________  

6. Find all points on the graph of \( z = xy^3 + 10y^{-1} + 12 \) where the vector \( \mathbf{n} = \langle 16, -7, 2 \rangle \) is normal to the tangent plane.

point(s) on graph: ___________________________
Let $r$, $s$, and $t$ be independent parameters and suppose $x$, $y$, and $z$ are given by

\begin{align*}
  x &= 2r - 3s + t \\
  y &= 5r + 2s - 6t \\
  z &= -r + s
\end{align*}

Let $w = f(x, y, z)$ where $f$ is an arbitrary differentiable function. Calculate the sum

\[ A(r, s, t) = \frac{\partial w}{\partial r} + \frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} \]

Write your answer as a function of $r$, $s$, and $t$. Simplify as much as possible.
(Since $f$ is arbitrary, your answer may still contain the symbol $f$ or related symbols. But you must write your answer as a function of $r$, $s$, and $t$.)

\[ A(r, s, t) = \]
8. Note: This problem continues onto the next page.

Let \( f(x, y) = x^2 + y^2 - xy - 6x \)

(a) Find the critical point of \( f \) and the corresponding critical value. Then classify it as a local minimum, local maximum, or neither (saddle).

- Critical point: __________________
- Critical value: ________________
- Classification: ________________
Recall $f(x, y) = x^2 + y^2 - xy - 6x$. Let $S$ be the square \{(x, y) : 0 \leq x \leq 6, 0 \leq y \leq 6\}.

(b) Find the minimum and maximum values of $f$ on each of the four edges of $S$. Then determine the global extreme values of $f$ on $S$. Fill in the table below as you work.

<table>
<thead>
<tr>
<th>edge of $S$</th>
<th>bottom edge</th>
<th>right edge</th>
<th>top edge</th>
<th>left edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum value of $f$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
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global minimum value: ______________________

global maximum value: ______________________
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