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This exam has 8 questions, printed in 2 booklets, for a total of 100 points.

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1. Let \( \mathbf{v} = \mathbf{i} - \lambda \mathbf{j} + 3 \mathbf{k} \) and \( \mathbf{w} = \lambda \mathbf{i} - 4 \mathbf{j} + \mu \mathbf{k} \), where \( \lambda \) and \( \mu \) are real numbers.

(a) Calculate \( \|\mathbf{v}\| \) in terms of \( \lambda \) and \( \mu \).

\[ \|\mathbf{v}\| = \] 

(b) Calculate \( \mathbf{v} \cdot \mathbf{w} \) and \( \mathbf{v} \times \mathbf{w} \) in terms of \( \lambda \) and \( \mu \).

\[ \mathbf{v} \cdot \mathbf{w} = \] 
\[ \mathbf{v} \times \mathbf{w} = \] 

(c) Use your answer to part (b) to find the values of \( \lambda \) and \( \mu \) such that \( \mathbf{v} \) and \( \mathbf{w} \) are parallel. (Assume \( \lambda \) and \( \mu \) are positive real numbers.)

\[ \lambda = \] 
\[ \mu = \]
2. The lines $\ell_1$ and $\ell_2$ are given by the following parametrizations.

$$\ell_1: \quad \mathbf{r}_1(t) = \langle -2, -1, 4 \rangle + t \langle -5, 5, 1 \rangle$$

$$\ell_2: \quad \mathbf{r}_2(t) = \langle 0, -10, 10 \rangle + t \langle 3, 4, -7 \rangle$$

(a) Show that $\ell_1$ and $\ell_2$ intersect and find the point of intersection. Is this point also a collision point? Explain.

point of intersection: 

collision point? (yes/no): 

(b) Find an equation of the plane $\mathcal{P}$ that contains both $\ell_1$ and $\ell_2$.

equation of $\mathcal{P}$: 

3. Consider the curve $\mathcal{C}$ with parametrization

$$\mathbf{r}(t) = (t^2 - 3)i + (3t^2 + 5)j + \frac{2}{3}t^3k, \quad t \geq 0$$

(a) Find the length of $\mathcal{C}$ over the interval $0 \leq t \leq 1$.

length: ______________

(b) Find the curvature of $\mathcal{C}$ at the point $\mathbf{r}(1)$. You may use the formula

$$\kappa(t) = \frac{||\mathbf{r}''(t) \times \mathbf{r}'(t)||}{||\mathbf{r}'(t)||^3}$$

curvature: ______________
4. Assume that the positive $x$-axis points East and the positive $y$-axis points North. Suppose you are hiking on a terrain modeled by the equation $z = \sqrt{3}xy - 2x^2 - 1$ and you are standing at the point $(1, \sqrt{3}, 0)$.

(a) Determine the angle of inclination you would encounter if you headed due West.

angle of inclination: ________________

(b) Determine the steepest slope you could encounter from your position and the compass direction measured in degrees anticlockwise from East that you would head to realize this steepest slope.

steepest slope: ________________
compass direction: ________________

(c) In what direction should you head to encounter no change in elevation? Give your answer as an angle measured in degrees anticlockwise from East.

compass direction: ________________
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5. Calculate the following limit or show that it does not exist.

\[
\lim_{(x, y) \to (0, 0)} \left( \frac{xy}{3x^2 + 2y^2} \right)
\]

value of limit: __________________

6. Find all points on the graph of \( z = xy^3 + 10y^{-1} + 12 \) where the vector \( \mathbf{n} = (16, -7, 2) \) is normal to the tangent plane.

point(s) on graph: __________________________
7. Let $r$, $s$, and $t$ be independent parameters and suppose $x$, $y$, and $z$ are given by

\begin{align*}
x &= 2r - 3s + t \\
y &= 5r + 2s - 6t \\
z &= -r + s
\end{align*}

Let $w = f(x, y, z)$ where $f$ is an arbitrary differentiable function. Calculate the sum

$$A(r, s, t) = \frac{\partial w}{\partial r} + \frac{\partial w}{\partial s} + \frac{\partial w}{\partial t}$$

Write your answer as a function of $r$, $s$, and $t$. Simplify as much as possible. 

(Since $f$ is arbitrary, your answer may still contain the symbol $f$ or related symbols. But you must write your answer as a function of $r$, $s$, and $t$.)

$$A(r, s, t) = \text{________________________}$$
8. Note: This problem continues onto the next page.

Let \( f(x, y) = x^2 + y^2 - xy - 6x \)

(a) Find the critical point of \( f \) and the corresponding critical value. Then classify it as a local minimum, local maximum, or neither (saddle).

critical point: 

critical value: 

classification: 

Recall $f(x, y) = x^2 + y^2 - xy - 6x$. Let $S$ be the square $\{(x, y) : 0 \leq x \leq 6, 0 \leq y \leq 6\}$.

(b) Find the minimum and maximum values of $f$ on each of the four edges of $S$. Then determine the global extreme values of $f$ on $S$. Fill in the table below as you work.

<table>
<thead>
<tr>
<th>edge of $S$</th>
<th>bottom edge</th>
<th>right edge</th>
<th>top edge</th>
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<tbody>
<tr>
<td>minimum value of $f$</td>
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 global minimum value: __________________________

 global maximum value: __________________________
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