1. Let $V$ be the volume of a right circular cone of height 12 whose base is a circle of radius 3.

   (a) Use similar triangles to find the area of a horizontal cross section at a height $y$.
   
   (b) Calculate $V$ by integrating the cross-sectional area.

2. Let $R$ be the region bounded by the coordinate axes and the line $y = 6 - 3x$. Let $S$ be the solid whose base is $R$ and whose cross-sections perpendicular to the $x$-axis are squares whose bases lie in the $xy$-plane. Find the volume of $S$.

3. Let $R$ be the region bounded by the circle $x^2 + y^2 = 9$. Let $S$ be the solid whose base is $R$ and whose cross-sections perpendicular to the $y$-axis are right isosceles triangles whose hypotenuses lie in the $xy$-plane. Find the volume of $S$.

4. Let $R$ be the region bounded by the curves $y = e^{2x}$, $y = e^{-3x}$, and $x = \ln(5)$. Let $S$ be the solid whose base is $R$ and whose cross-sections perpendicular to the $x$-axis are half-discs whose diameters lie in the $xy$-plane. Find the volume of $S$.

5. Let $R$ be the region bounded by the $x$-axis and the curves $y = 2\sqrt{x - 1}$ and $y = x$. Let $S$ be the solid whose base is $R$ and whose cross-sections perpendicular to the $y$-axis are equilateral triangles whose bases lie in the $xy$-plane. Find the volume of $S$.

6. The population density $r$ miles from the center of a certain city is

   \[ \rho(r) = 30,000e^{-0.01r^2} \]

   people per square mile. How many people live between 2 and 5 miles from the city center?

7. What is the average value of $y = \frac{\sin(\pi/x)}{x^2}$ on the interval $[1, 2]$?

8. Let $\rho(u) = \frac{u^2}{u^2 + 1}$.

   (a) A thin rod of total length 10 cm has linear density $\rho(x)$ g/cm, where $x$ is the distance (in centimeters) from one end of the rod. What is the total mass of the rod?

   (b) A thin circular plate with radius 10 cm has radial density $\rho(r)$ g/cm$^2$, where $r$ is the distance (in centimeters) from the center of the plate. What is the total mass of the plate?
1. See lecture notes.

2. \[ V = \int_0^2 (6 - 3x)^2 \, dx = 24 \]

3. \[ V = \int_{-3}^3 \frac{1}{4} \left( 2 \sqrt{9 - y^2} \right)^2 \, dy = \int_{-3}^3 (9 - y^2) \, dy = 36 \]

4. \[ A = \pi (\frac{d}{2})^2 = \frac{\pi d^2}{4} \]

\[ V = \int_0^{\ln(5)} \frac{\pi}{4} \left( e^{2x} - e^{-3x} \right)^2 \, dx = \frac{603,776 \pi}{15,625} \]

5. \[ V = \frac{\sqrt{3}}{4} \int_0^2 \left( y^{\frac{3}{4}} - y + 1 \right)^2 \, dy = \frac{\sqrt{3}}{10} \]
6) \[ P = 60,000 \pi \int_2^5 re^{-0.01r^2} \, dr = \]
\[ = 3,000,000 \pi e^{-1/4} \left( e^{21/100} - 1 \right) \]

7) \[ M = \frac{1}{2-1} \int_1^2 \frac{\sin(\pi/x)}{x^2} \, dx = \frac{1}{\pi} \]

8) (a) \[ m = \int_0^{10} \frac{u^2}{u^2 + 1} \, du = \int_0^{10} \left( 1 - \frac{1}{u^2 + 1} \right) \, du \]
\[ = \left( u - \tan^{-1}(u) \right) \bigg|_0^{10} = 10 - \tan^{-1}(10) \]

(b) \[ m = 2\pi \int_0^{10} \frac{r^3}{r^2 + 1} \, dr \]
\[ = 2\pi \int_0^{10} \left( r - \frac{r}{r^2 + 1} \right) \, dr \]
\[ = 2\pi \left( \frac{r^2}{2} - \frac{1}{2} \ln(10) \right) \bigg|_0^{10} \]
\[ = 2\pi \left( 50 - \frac{1}{2} \ln(101) \right) \]