1. For each series, determine whether it converges or diverges. For parts (c) and (d), your answer may depend on the values of \( p \) or \( x \).

(a) \[ \sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!} \]

(b) \[ \sum_{n=1}^{\infty} \left( 1 + \frac{3}{n} \right)^{-n^2} \]

(c) \[ \sum_{n=1}^{\infty} \frac{n^p}{n!} \text{ \( (p \) constant)} \]

(d) \[ \sum_{n=1}^{\infty} \frac{x^n}{n!} \text{ \( (x \) constant)} \]

(e) \[ \sum_{n=0}^{\infty} \frac{n!}{100^n} \]

(f) \[ \sum_{n=1}^{\infty} \left( \frac{n}{3n + 1} \right)^n \]

(g) \[ \sum_{n=0}^{\infty} \frac{(2n)!(2n)!}{n!(3n)!} \]

(h) \[ \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n - 1)}{3^n n!} \]

(i) \[ \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n - 1)}{2^n n!} \]

2. Consider the series \( S(a) = \sum_{n=1}^{\infty} n!a^n n^{-n} \)

(a) Use Ratio Test to show that \( S(a) \) converges if \( |a| < e \) and diverges if \( |a| > e \). The test remains inconclusive for \( a = -e \) or \( a = e \).

(b) Determine whether \( S(e) \) converges.

(c) Determine whether \( S(-e) \) converges.

**Hint:** For parts (b) and (c), obviously you cannot use Ratio Test or Root Test since both tests are inconclusive. Instead, recall from lecture that if \( n \) is large enough then there is some constant \( C > 1 \) such that

\[ n! \sim Cn^{n+1/2}e^{-n} \]

Note that this precisely means that

\[ \lim_{n \to \infty} \left( \frac{n!}{n^{n+1/2}e^{-n}} \right) = C \]

This is known as Stirling’s approximation and can be proved using Calculus I. We need more advanced mathematics to show that \( C = \sqrt{2\pi} \).