1. The following three facts are very useful when dealing with series whose terms involve sine or cosine. Even if you do not prove the following facts, you should use them freely in exercises with series.

(a) Suppose $c > 0$. Explain why the sequence $a_n = \sin(c/n)$ is eventually positive. That is, there is some integer $N$ for which $a_n > 0$ for all $n > N$.

(b) Same as part (a) but for $a_n = \cos(c/n)$.

(c) Show that $|\sin(x)| \leq |x|$ for all $x$. Hint: Apply the mean value theorem to $f(x) = \sin(x)$ on the interval $[0, b]$ for $b > 0$.

2. For each series, determine whether it converges or diverges. Be sure to quote any theorem (convergence test) you use by name and fully justify its use.

(a) $\sum_{n=1}^{\infty} n \tan^{-1} \left( \frac{\pi}{n} \right)$

(b) $\sum_{n=1}^{\infty} \frac{3^n + 7^n}{5^n + 6^n}$

(c) $\sum_{n=1}^{\infty} \frac{4^n - 3^n}{8^n + 4n}$

(d) $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$

(e) $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

(f) $\sum_{n=2}^{\infty} \frac{\ln(n)}{\sqrt{n}}$

(g) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^3}$

(h) $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)^2}$

(i) $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln(n)}$

(j) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n \ln(n)^4}}$

(k) $\sum_{n=1}^{\infty} \frac{3n + 5}{\sqrt{n^3 + 8n + 1}}$

(l) $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^{1/4}}$

(m) $\sum_{n=1}^{\infty} \left( 1 - \frac{2}{n} \right)^n$