1. Determine the radius and interval of convergence of the following series.

\[ \sum_{n=1}^{\infty} \frac{n}{n^2 + 1} (x - 3)^n \]

**Solution**
We first use Ratio Test.

\[ \rho = \lim_{n \to \infty} \left| \frac{(n + 1)(x - 3)^{n+1}}{(n + 1)^2 + 1} \cdot \frac{n^2 + 1}{n(x - 3)^n} \right| = \lim_{n \to \infty} \left| \frac{n+1}{n} \cdot \frac{n^2 + 1}{n^2 + 2n + 2} \right| = |x - 3| \]

Hence the series converges for \( |x - 3| < 1 \) and diverges for \( |x - 3| > 1 \). The radius of convergence is \( R = 1 \) and the interval convergence is \((2, 4)\) (possibly with one or both endpoints). Now we check the endpoints.

For \( x = 4 \), we consider the series \( \sum_{n=1}^{\infty} \frac{n}{n^2 + 1} \). The series \( \sum_{n=2}^{\infty} \frac{1}{n} \) diverges by \( p \)-test \( (p = 1 \leq 1) \) and \( \lim_{n \to \infty} \frac{n^2 + 1}{n} = \lim_{n \to \infty} \frac{n^2}{n} = 1 \). Hence by the limit comparison test, the series \( \sum_{n=1}^{\infty} \frac{n}{n^2 + 1} \) diverges.

For \( x = -2 \), we consider the series \( \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1} \). Let \( a_n = \frac{n}{n^2 + 1} \). Then \( \{a_n\} \) is positive, is decreasing, and has limit 0. Hence the series \( \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1} \) converges by alternating series test.

In summary, the interval of convergence is \([-2, 4)\).

2. Find the Maclaurin series of \( f(x) = x^3 \sin(2x) \). Write out the first four non-zero terms and also write the general term of the series.

**Solution**
We start with the Maclaurin series for \( \sin(x) \), compose with \( 2x \), and multiply by \( x^3 \).

\[ \sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \]

\[ \sin(2x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!} = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!} + \cdots \]

\[ = \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n+1} x^{2n+1}}{(2n+1)!} = 2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots \]

\[ x^3 \sin(2x) = \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n+1} x^{2n+4}}{(2n+1)!} = 2x^4 - \frac{8x^7}{3!} + \frac{32x^{10}}{5!} - \frac{128x^{13}}{7!} + \cdots \]