6.1: Area Between Curves

\[ A = \int_{a}^{b} (f(x) - g(x)) \, dx \quad \text{or} \quad A = \int_{c}^{d} (f(y) - g(y)) \, dy \]

6.2: Volume, Density, Average Value

- **Average Value of** \( f(x) \) **on** \([a, b] \):

  \[ M = \frac{1}{b-a} \int_{a}^{b} f(x) \, dx \]

- **Total mass** (charge, population, etc.) **given linear density** \( \lambda(x) \):

  \[ m = \int_{a}^{b} \lambda(x) \, dx \]

- **Total mass** (charge, population, etc.) **given radial density** \( \sigma(r) \):
\[ m = \int_{r_1}^{r_2} 2\pi r v(r) \, dr \]

- Volume of solid for which cross-sections perpendicular to x-axis have area \( A(x) \) at coordinate \( x \).

\[ V = \int_{a}^{b} A(x) \, dx \]

### 6.3: Volumes of Revolution

- Method of Washers:

\[ V = \int_{a}^{b} \pi \left( (\text{Outer})^2 - (\text{Inner})^2 \right) \, dh \]

- Volume by slicing: cross section is perpendicular to axis of rotation
  - For vertical slice, \( dh = dx \)
  - For horizontal slice, \( dh = dy \)

### 6.4: Method of Cylindrical Shells

\[ V = \int_{a}^{b} 2\pi r h \, dr \]
• Volume obtained by rotating rectangular strips to form nested shells.

• Strip is parallel to axis of rotation

• \( F \): average radius of shell (distance from rotation axis to strip)

• \( h \): height of shell (length of strip)

• For vertical strip, \( dr = dx \)

For horizontal strip, \( dr = dy \)

8.1: Arc Length and Surface Area

• Length of curve \( y = f(x) \) over \([a, b]\).

\[
 s = \int_a^b \sqrt{1 + (f'(x))^2} \, dx
\]

• Length of curve \( x = f(y) \) over \([c, d]\).

\[
 s = \int_c^d \sqrt{1 + (f'(y))^2} \, dy
\]

• Arc length differential

\[
 ds = \sqrt{(dx)^2 + (dy)^2}
\]
\[ = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \]
\[ = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy \]

• Area of surface obtained by rotating the curve \( y = f(x) \) (\( a \leq x \leq b \)) about the \( x \)-axis:
\[ A = \int_{a}^{b} 2\pi f(x) \sqrt{1 + f'(x)^2} \, dx \]

• General surface area formula:
\[ A = \int_{a}^{b} 2\pi \overline{r} \, ds \]

• Area obtained by rotating conical bands about axis of rotation.

• \( \overline{r} \): average radius of band (distance from rotation axis to strip)

• \( ds \): arc length differential