1. Let $C$ be the curve that consists of the graph of $y = x^3$ over the interval $[0, 4]$.

   (a) Find but do not evaluate an integral whose value is the arc length of $C$.

   (b) Let $S$ be the surface obtained by rotating $C$ about the $x$-axis. Find and evaluate an integral whose value is the area of $S$.

2. For each of the following sequences, determine whether it converges or diverges. If the sequence converges, find its limit. If the sequence diverges or if there is not enough information to answer the question, explain why.

   (a) $a_n = \frac{(-1)^n}{\sqrt{n}}$

   (b) $b_n = \ln\left(\frac{3n^2 + 4n - 5}{7n^2 - 3n + 6}\right)$

   (c) $c_n = \left(1 + \frac{2}{n}\right)^{3n}$

   (d) $\{d_n\}$, a sequence such that $\sum_{n=1}^{\infty} d_n = 1$

3. For each of the following series, determine whether it converges or diverges. If you use a convergence test, you must identify the test by name and show that the conditions for applying the test are satisfied. If you have no justification, you will receive zero credit!

   (a) $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$

   (b) $\sum_{n=1}^{\infty} \frac{n^3}{\sqrt{2n^8 + 3n + 1}}$

   (c) $\sum_{n=1}^{\infty} ne^{-n^2}$

   (d) $\sum_{n=1}^{\infty} a_n$, a series whose partial sums are $S_N = \frac{1}{N}$

4. For each of the following series, either calculate its sum or explain why it diverges.

   (a) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

   (b) $\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$
5. Let \( S \) be the series
\[
S = 1 - \frac{1}{2^{1/2}} + \frac{1}{3^{1/2}} - \frac{1}{4^{1/2}} + \frac{1}{5^{1/2}} - \frac{1}{6^{1/2}} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/2}}
\]

(a) Show that \( S \) converges.

(b) Suppose you estimate \( S \) by summing the first \( N \) terms of this series. How large should \( N \) be to guarantee that your estimate has an absolute error of at most \( 10^{-2} \)? Justify your answer.

6. Suppose \( f(x) \) is defined via the following power series.
\[
f(x) = \sum_{n=0}^{\infty} \frac{(-8)^n}{(n+1)^2} (x-2)^{3n} = 1 - 2(x-2)^3 + \frac{64}{9} (x-2)^6 - 32(x-2)^9 + \cdots
\]

(a) Find the radius of convergence of this series.

(b) Find the interval of convergence of this series.

7. Find the Maclaurin series for \( f(x) = \ln(1+3x) \) and its interval of convergence.

8. (The parts of this question are not related.)

(a) Suppose \( f \) is three-times differentiable, and you know only the following:
\[
f(5) = 2, \quad f'(5) = -\frac{1}{4}, \quad f''(5) = 0, \quad f'''(5) = 10
\]
Find \( T_3(x) \), the third degree Taylor polynomial of \( f \) centered at \( c = 5 \).

(b) Find the Maclaurin series of \( g(x) = x^3 \sin(2x^2) \). (You should write at least the first three non-zero terms of your series.) Then use your series to calculate \( g^{(9)}(0) \).