Book #1 of 1

Name: ____________________________

ID# (last 4 digits): ____________________ Section: __________________________

Unless stated otherwise, you must show all work clearly using proper notation and explain your reasoning in English where appropriate. Answers must be justified using techniques that have been taught in this course, and answers without such justification may receive less than full credit – or no credit at all – even if the answer is correct.

This exam is closed book. Calculators, electronic devices, notes, books, formula sheets, and other outside materials are not allowed. Phones must be turned off and put away.

Unless otherwise stated, give exact answers: e.g., write $\pi$ and $\sqrt{2}$ instead of 3.14 and 1.41. However, when an expression simplifies to a well-known value, you must use that value. For example, you must write 1 instead of $e^0$, and you must write $\frac{1}{2}$ instead of $\cos\left(\frac{\pi}{3}\right)$.

Any expression with an inverse trigonometric function nested within a trigonometric function (e.g., $\cos(\sin^{-1}(x))$) must be simplified so neither special function appears. The same rule applies for hyperbolic functions and their inverses.

This exam has 8 questions, printed in 1 booklet(s), for a total of 100 points.

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1. Let $C$ be the curve that consists of the graph of $y = x^3$ over the interval $[0,4]$.

(a) **Find but do not evaluate** an integral whose value is the arc length of $C$.

arc length: __________

(b) Let $S$ be the surface obtained by rotating $C$ about the $x$-axis. **Find and evaluate** an integral whose value is the area of $S$.

surface area: __________
2. Note: This problem continues onto the next page.

For each of the following sequences, determine whether it converges or diverges. If the sequence converges, find its limit. If the sequence diverges or if there is not enough information to answer the question, explain why.

(a) \( a_n = \frac{(-1)^n}{\sqrt{n}} \)

(b) \( b_n = \ln \left( \frac{3n^2 + 4n - 5}{7n^2 - 3n + 6} \right) \)
Note: This is a continuation of the problem on the previous page.

For each of the following sequences, determine whether it converges or diverges. If the sequence converges, find its limit. If the sequence diverges or if there is not enough information to answer the question, explain why.

(c) \( c_n = \left(1 + \frac{2}{n}\right)^{3n} \)

(d) \( \{d_n\} \), a sequence such that \( \sum_{n=1}^{\infty} d_n = 1 \)
3. **Note: This problem continues onto the next page.**

For each of the following *series*, determine whether it converges or diverges. If you use a convergence test, you must identify the test by name and show that the conditions for applying the test are satisfied. *If you have no justification, you will receive zero credit!*

(a) \( \sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!} \)

(b) \( \sum_{n=1}^{\infty} \frac{n^3}{\sqrt{2n^8 + 3n + 1}} \)
Note: This is a continuation of the problem on the previous page.

For each of the following series, determine whether it converges or diverges. If you use a convergence test, you must identify the test by name and show that the conditions for applying the test are satisfied. If you have no justification, you will receive zero credit!

5 pts

(c) \( \sum_{n=1}^{\infty} ne^{-n^2} \)

5 pts

(d) \( \sum_{n=1}^{\infty} a_n \), a series whose partial sums are \( S_N = \frac{1}{N} \)
4. For each of the following series, either calculate its sum or explain why it diverges.

(a) \[ \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+2} \right) \]

(b) \[ \sum_{n=0}^{\infty} \frac{(-2)^n}{n!} \]
5. Let $S$ be the series

$$S = 1 - \frac{1}{2^{1/2}} + \frac{1}{3^{1/2}} - \frac{1}{4^{1/2}} + \frac{1}{5^{1/2}} - \frac{1}{6^{1/2}} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/2}}$$

(a) Show that $S$ converges.

(b) Suppose you estimate $S$ by summing the first $N$ terms of this series. How large should $N$ be to guarantee that your estimate has an absolute error of at most $10^{-2}$? Justify your answer.
6. Suppose \( f(x) \) is defined via the following power series.

\[
f(x) = \sum_{n=0}^{\infty} \frac{(-8)^n}{(n+1)^2} (x - 2)^{3n} = 1 - 2(x - 2)^3 + \frac{64}{9} (x - 2)^6 - 32(x - 2)^9 + \cdots
\]

(a) Find the radius of convergence of this series.

(b) Find the interval of convergence of this series.
7. Find the Maclaurin series for \( f(x) = \ln(1 + 3x) \) and its interval of convergence.

Maclaurin series: ________________

interval of convergence: ________________
8. (The parts of this question are not related.)

5 pts (a) Suppose $f$ is three-times differentiable, and you know only the following:

\[ f(5) = 2, \quad f'(5) = -\frac{1}{4}, \quad f''(5) = 0, \quad f'''(5) = 10 \]

Find $T_3(x)$, the third degree Taylor polynomial of $f$ centered at $c = 5$.

5 pts (b) Find the Maclaurin series of $g(x) = x^3 \sin(2x^2)$. (You should write at least the first three non-zero terms of your series.) Then use your series to calculate $g^{(6)}(0)$. 
This page is for scratch work. Do not detach this sheet.