1. Find the absolute maximum and absolute minimum values of \( f(x) = \frac{10x}{x^2 + 1} \) on the interval \([0, 2]\).

**Solution**

The function \( f \) is differentiable on all intervals. So the only critical numbers are those \( x \)-values such that \( f'(x) = 0 \).

\[
f'(x) = \frac{(x^2 + 1)(10) - (10x)(2x)}{(x^2 + 1)^2}
\]

\[
0 = \frac{10 - 10x^2}{(x^2 + 1)^2}
\]

\[
0 = 10 - 10x^2
\]

The only solution to the equation \( 1 - x^2 = 0 \) in the interval \([0, 2]\) is \( x = 1 \). Checking the critical value and the endpoint values gives the following.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = \frac{10x}{x^2 + 1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

The maximum value of \( f \) on the interval \([0, 2]\) is 5 and the minimum value is 0.

2. For each part, determine whether the hypotheses of mean value theorem are satisfied by \( f \) on the given interval. Explain your answer.

(a) \( f(x) = \frac{x^2 - 5x}{x^3 + 1} \) on \([-2, 5]\)

(b) \( f(x) = |4x - 20| + \sin(4x) \) on \([-\pi, \pi]\)

**Solution**

(a) The function \( f \) is discontinuous (in fact, undefined) at \( x = -1 \). Since \(-1 \in [-2, 5]\), the function \( f \) does not satisfy the hypotheses of the MVT on \([-2, 5]\).

(b) The function \( f \) is continuous for all \( x \), but not differentiable at \( x = 5 \). Since \( 5 \notin (-\pi, \pi) \), the function \( f \) does satisfy the hypotheses of the MVT on \([-\pi, \pi]\).