1. Consider the graph for the distance from the origin versus time for a certain particle. Distance is measured in kilometers and time is measured in hours. You may assume that the distance of the particle from the origin is 150 km for all \( t > 3 \).

Write your answer using interval notation where appropriate.

(a) When is the velocity of the particle positive?
(b) When is the particle at rest?
(c) When is the particle’s velocity decreasing?
(d) Estimate the velocity of the particle at \( t = 0.5 \). You must include correct units as part of your answer.

Solution

(a) The velocity is positive at points where the tangent line has positive slope. This occurs for \( t \in (0, 1) \cup (2, 3) \).

(b) The particle is at rest at any points where the tangent line is horizontal. This occurs for \( t \in (1, 1.5) \cup \{2\} \cup (3, \infty) \). The value \( t = 0 \) may or may not be included.

(c) The velocity is decreasing where the tangent lines have decreasing slope, or where the graph is curving downward. This occurs for \( t \in (0.5, 1) \cup (1.5, 1.75) \cup (2.5, 3) \).

(d) The velocity of the particle at \( t = 0.5 \) is the slope of the tangent line at \( t = 0.5 \). We may estimate that this tangent line passes through the points \((0.5, 50)\) and \((0.75, 100)\). The slope of this line is \( m = \frac{100 - 50}{0.75 - 0.5} = 200 \). Thus the velocity is 200 km/h.

2. Let \( g(x) = \frac{2x - 1}{5 - 3x} \). Find an equation of the tangent line at \( x = 2 \).
Solution
First we compute $g'(x)$ by quotient rule.

$$g'(x) = \frac{(5 - 3x)(2) - (2x - 1)(-3)}{(5 - 3x)^2} = \frac{7}{(5 - 3x)^2}$$

Now observe that $g(2) = -3$ and $g'(2) = 7$. Hence the tangent line has slope $-3$ and passes through the point $(2, 7)$. So an equation of the tangent line is

$$y - 7 = -3(x - 2)$$