1. Evaluate \( \frac{d}{dx} \left( \int_{\sin(x)}^{\pi/2} \tan^{-1}(e^t + 4) \, dt \right) \).

**Solution**

Use the fundamental theorem of calculus (part II) with the chain rule.

\[
\frac{d}{dx} \left( \int_{\sin(x)}^{\pi/2} \tan^{-1}(e^t + 4) \, dt \right) = -\tan^{-1}(e^{\sin(x)} + 4) \cdot \cos(x)
\]

2. Evaluate \( \int_{e^2}^{e^5} \frac{1}{x \ln(x)^2} \, dx \).

**Solution**

Use the substitution \( u = \ln(x) \) (whence \( du = \frac{1}{x} \, dx \)) and the fundamental theorem of calculus (part I).

\[
\int_{e^2}^{e^5} \frac{1}{x \ln(x)^2} \, dx = \int_2^5 \frac{1}{u^2} \, du = \left( -\frac{1}{u} \right) \bigg|_2^5 = -\frac{1}{5} - -\frac{1}{2} = \frac{3}{10}
\]

3. A particle moves in a straight line with velocity \( v(t) = t^2 - 4t \).

   (a) Find the displacement of the particle during the time interval \( 0 \leq t \leq 5 \).
   (b) Find the distance traveled by the particle during the time interval \( 0 \leq t \leq 5 \).

**Solution**

(a) The displacement of a particle is the integral of its velocity. Hence we have

\[
\Delta x = \int_0^5 (t^2 - 4t) \, dt = \left( \frac{t^3}{3} - 2t^2 \right) \bigg|_0^5 = \frac{125}{3} - 50 - 0 = -\frac{25}{3}
\]

(b) The distance of a particle is the integral of its speed (i.e., \( |v(t)| \)). First we find that \( v(t) = 0 \) when \( t = 0 \) or when \( t = 4 \). On the interval \( [0, 4] \) the velocity \( v(t) \) is negative, and on the interval \( [4, 5] \) the velocity \( v(t) \) is positive. Hence we have

\[
d = \int_0^5 |v(t)| \, dt = \int_0^4 |v(t)| \, dt + \int_4^5 |v(t)| \, dt = -\int_0^4 v(t) \, dt + \int_4^5 v(t) \, dt
\]
We now use fundamental theorem of calculus to evaluate each integral.

\[
\int_0^4 v(t) \, dt = \left( \frac{t^3}{3} - 2t^2 \right) \bigg|_0^4 = \frac{64}{3} - 32 = -\frac{32}{3}
\]

\[
\int_4^5 v(t) \, dt = \left( \frac{t^3}{3} - 2t^2 \right) \bigg|_4^5 = \left( \frac{125}{3} - 50 \right) - \left( \frac{64}{3} - 32 \right) = \frac{7}{3}
\]

Hence the total distance is \( d = -\left( \frac{32}{3} \right) + \frac{7}{3} = 13 \).