Book #1 of 1

Name: ________________________________

ID# (last 4 digits): ________________________________

Section: ________________________________

- Please explain your answers clearly and show all work in the space provided. I reserve the right to give no credit for a response with no work even if the final answer is correct.

- You are not allowed the use of any calculator, unapproved formula sheet, or electronic device.

- All electronic devices must be turned off.

- Unless noted otherwise, all final answers should be exact.

- This exam has 8 questions, for a total of 100 points.

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1. Suppose $g(x)$ is differentiable for all real numbers.

(a) Write down the precise definition of $g'(3)$.

(b) Explain the relationship between $g'(3)$ and the line tangent to the graph of $y = g(x)$ at $x = 3$. 

3 pts
2. On the set of axes provided below, sketch the graph of a function $f(x)$ that satisfies all of the following properties.

- the domain of $f$ is all real numbers
- $\lim_{x \to -4^+} f(x) = f(-4)$ and $f$ is not continuous at $x = -4$
- $\lim_{x \to 1} f(x)$ exists and $f$ is not continuous at $x = 1$
- $f$ is continuous and not differentiable at $x = 5$
3. For each part, calculate $f'(x)$.

After calculating the derivative, do not simplify your answer.

8 pts  (a) $f(x) = \sin(7xe^{-3x})$

8 pts  (b) $f(x) = \sqrt{\frac{2 \ln(x)}{\tan(3x) - \tan(3)}}$
4. Find an equation of the line tangent to the curve

\[ \sin(x - y) = 4e^{xy} - 4e^9 \]

at the point (3, 3).
5. For each limit, calculate the value or show that it does not exist. If the limit is $+\infty$ or $-\infty$, that should be your answer instead of “does not exist”. Show all work.

(a) $\lim_{t \to 3^-} \frac{8 \cos(\pi t) + t}{t^2 - 9}$

(b) $\lim_{h \to 4} \frac{(h + 6)^2 - 25h}{h - 4}$
6. Find all horizontal asymptotes of the function

\[ f(x) = \frac{\sqrt{16x^4 + 7x + 5}}{3x - 8} \]
7. Note: This problem continues onto the next page.

Each part of this question refers to the function \( f \) below, where \( a \) and \( b \) are constants.

\[
f(x) = \begin{cases} 
\frac{2x}{\sin(ax)} & , \quad x < 0 \\
x - 4 & , \quad 0 \leq x < 5 \\
b & , \quad x = 5 \\
\frac{4 - \sqrt{3x + 1}}{x - 5} & , \quad x > 5 
\end{cases}
\]

For each of the following parts, you must give a full, clear justification for your answer. You must use proper methods taught in this course.

9 pts

(a) Find the value of the constant \( a \) so that \( f \) is continuous at \( x = 0 \). If this is not possible, explain why.
Note: This a continuation of the problem on the previous page.

\[ f(x) = \begin{cases} 
\frac{2x}{\sin(ax)} , & x < 0 \\
x - 4 , & 0 \leq x < 5 \\
b , & x = 5 \\
\frac{4 - \sqrt{3x + 1}}{x - 5} , & x > 5 
\end{cases} \]

(b) Find the value of the constant \( b \) so that \( f \) is continuous at \( x = 5 \). If this is not possible, explain why.
8. Consider the claim that \( \lim_{x \to 4} (8x + 1) = 33 \).

(a) Write down the precise definition of limit (using \( \epsilon \) and \( \delta \), as it applies to the this particular limit). Your statement should begin with “For each \( \epsilon > 0 \)...”.

(b) Using the precise definition of limit, prove that the claim is true. You must write a formal proof with appropriate mathematics and complete English sentences.
This page is for scratch work. Do not detach this sheet.
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